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ON THE

CONTRACTED LIQUID VEIN

AFFECTING THE PRESENT THEORY OF THE

SCIENCE OF HYDRAULICS,

BY

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DEPARTMENT PUBLIC WORKS, CANADA.

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ON THE CONTRACTED LIQUID VEIN

AFFECTING THE PRESENT THEORY OF THE SCIENCE OF HYDRAULICS

BY R. STECKEL, ASSISTANT ENGINEER,
DEPARTMENT OF PUBLIC WORKS, CANADA.

1883-84.

INTRODUCTION.

It has been proved in the most conclusive manner, a full century ago, by the celebrated Italian philosopher, Lorgna, founder of the "Societa Italiana," in the first chapter of his "Phisico Mathematical Theory of the Motion of Liquids issuing from Orifices in Reservoirs,"* and by other scientists, that the contracted fluid vein issuing from an orifice in the side or bottom of a reservoir constantly kept full of water, does not acquire its *vis viva* or living force by reason of the actual descent of the liquid particles, from the surface through the orifice. Yet, for the want of a sound theory, consistent with the results of experiment, respecting the formation of the liquid contracted vein, we are up to this day compelled, in the absence of any other alternative, to consider all liquid jets or veins in the light of bodies falling, in each case, freely through a space equal to the height of the liquid surface above the centre of the orifice, according to the universally accepted law of gravitation. We are also forced, chiefly for this reason, to introduce into all hydraulic computations, empirical coefficients of velocity, coefficients of contraction and coefficients of efflux or discharge, in addition to a variety of coefficients of friction and other resistances.

Some time ago I undertook a series of experiments, for the purpose of becoming practically acquainted with the leading hydraulic phenomena and thoroughly convinced of the truth of the commonly accepted laws by which the intricate and still imperfectly understood science of hydraulics is said to be governed. It is no more than might be expected, that such a prominent phenomenon as the contraction of the liquid vein at its exit from the orifice should attract a good share of attention on my part. I may state, however, that I was also incited to pursue deeply the investigation of this particular part of hydraulics by the perusal of such passages of the literature on the subject as the following, viz.:—

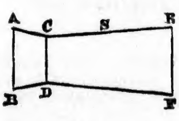


Fig. 1.

1. "By applying the general laws of motion to the "lateral fluid filaments of the stream which issues "through A B, it is found that they tend to describe a "curve which commences within the reservoir, for ex- "ample at A, and continues towards C S E. To deter- "mine the nature of this curve, it is requisite to know "and to combine together by calculation: the mutual con- "vergency of the fluid filaments in A B, the law of the "lateral communication of motion between the filaments "themselves and their divergent progression from C to E. "These combinations and calculations are perhaps "beyond the utmost efforts of analysis. While the tube

* Vol. IV., Mem. della Societa Italiana. (See Appendix.)

" A D F E, possesses a different figure from this natural curve, the results of experi-
" ment will always differ more or less from the theory (1).

2. " Lorgna pretends that 0·472 a (a being the head) is the height which would " produce, in any heavy body, the velocity of efflux in the orifice, and that the con- " tracted vein is nothing else than the continuation of the Newtonian Cataract: he " supports this proposition by computations deduced from the mutual action of the " particles of the fluid contained in the vessel. But after having seen the failures of " the greatest geometers on this very subject, we ought to mistrust all these demon- " strations founded on mechanical principles very true in themselves, but " of which the application to an infinity of bodies, which move and are pressed in " every direction, becomes extremely difficult, if not impossible." (2)

3. " So long as we have no more accurate knowledge of the law of con- " traction of the stream, we can assume that the stream flowing through a circular " orifice, forms a solid of rotation whose surface is generated by the revolution of the " arc of a circle about the axis of the stream. (3)

4. " It has been latterly asserted in a Blue-Book that theoretically $V_d = \frac{2}{3}\sqrt{2gh}$, " V_d denoting the velocity in the plane of an orifice in a thin plate; h, the head of " water on this orifice, and g, the acceleration produced by gravity, per second. It " is not necessary here to combat this error, which confounds the discharge with its " velocity, and a single practical fact, applicable only to a thin plate, with a theoretical " principle. The experimental discharge approximates to $\frac{2}{3}\sqrt{2gh}$ multiplied by the " area of the orifice; but the theoretical velocity $\sqrt{2gh}$ always approximates to the " experimental velocity, or ·974 $\sqrt{2gh}$, obtained immediately outside the orifice, in " the *venâ contractâ*. It would be unnecessary to allude to this theory here, if it " were not supported and put forward by three engineers whose authority in prac- " tical questions may mislead others. *Vide* p. 4 of 'Brief Observations of Messrs. " 'Bidder, Hawksley and Bazalgette on the answers of the Government Referees on " 'the Metropolitan Main Drainage,' ordered by the House of Commons (London, " Eng.), to be printed 13th July, 1858."*

The first part of Lorgna's "Phisico Mathematical Theory of the Motion of Liquids issuing from Orifices in Reservoirs," especially, is well worth perusing. As the fourth volume of the memoirs of the "Italian Society," published in 1785, which contains this *savant's* original paper in extenso, is not easy of access for consultation, I have appended hereto a translation of the introduction and the two first chapters.

EXPERIMENTAL ENQUIRIES.

APPARATUS USED—MODE OF CONDUCTING EXPERIMENTS.

In order that the experimental data to which I shall have to refer hereafter, in support of theoretical deductions, may prove acceptable with some degree of confidence, it is indispensable that I should give a brief description of the apparatus made use of, and of the *modus operandi* followed by me for their determination.†

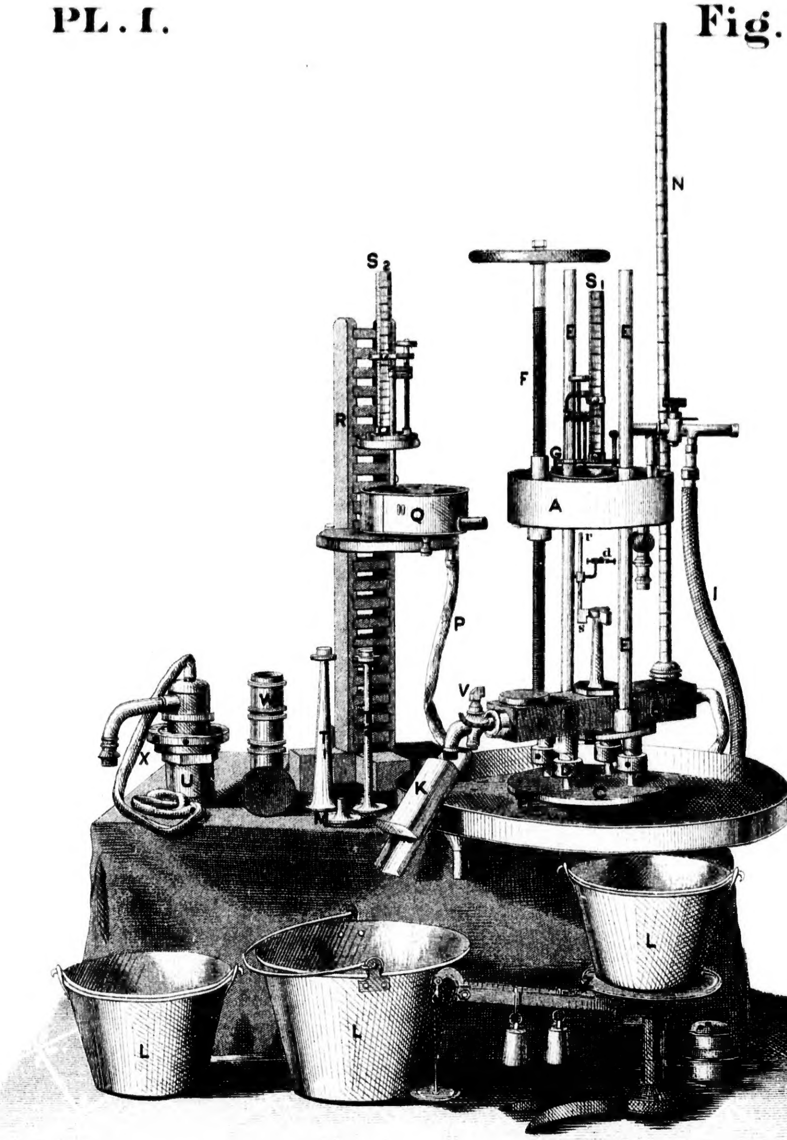
(1) and (2) See Tracts on Hydraulics, edited by Thomas Tredgold, London, 1826. Part II.—Experimental enquiries concerning the principle of lateral communication of motion in fluids applied to the explanation of various hydraulic phenomena, by Citizen Z. B. Venturi; translated from the French by W. Nicholson. Pages 145 and 177.

(3) See Weisbach's Mechanics of Engineering, page 822, vol. I. English translation by Coxe. Von Nostrand, New York.

* See Neville's Hydraulic Tables, coefficients and formulæ, second edition, p. 33.

† The apparatus shown in Fig. 2, including mouth-pieces, orifices, tubes, hook gauges and fittings was constructed for me by Mr. E. Chanteloup of Montreal, who executed the work with his accustomary ingenuity, precision and care.

PL. I. **Fig. 2.**



Apparatus *S. McLaughlin, Photo.*

used for making experiments.

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The reservoir of supply A—a brass cylinder 12 inches in diameter inside and some $3\frac{1}{2}$ inches high in the clear, was mounted on two horizontal, parallel circular plates B, C, respectively 10 and 12 inches in diameter, connected by four ball and socket jointed levelling screws D, by means of two guide rods E, and a feed screw F, about 3 feet in height, along which it could be raised or lowered at pleasure to any desired height above the upper plate B.

The orifice-plates O, mouth-pieces M, or tubes T, were screwed from below into a threaded ring in the center of the horizontal circular bottom of this reservoir A, and a brass stand G, carrying a hook-gauge and scale *S*., provided with vernier furnishing readings to within $\frac{1}{500}$ part of an inch, was screwed around the outer face of this interior ring-shaped projection, about $\frac{1}{2}$ inch in height, provided on the bottom of the said reservoir. A cylindrical, vertical, perforated partition of sheet copper, some 9 inches in diameter, and $3\frac{1}{2}$ inches high, was placed loosely in the centre of the reservoir A, for the purpose of counteracting such disturbances as might be produced by any appreciable centrifugal or other motion which the water might still have had after passing into the reservoir of supply proper, from a square tank situated in the garret of the building, through an inch supply pipe I, connected with a $\frac{3}{4}$ inch circular copper pipe laid on the bottom of the last mentioned reservoir, outside of the guide rods and screw, which was pierced on the outside by a number of small, round holes.*

The water was first received into a light trough K, of sheet copper, held by hand or otherwise secured in position, so as to be easily removable, when an experiment is finished, from underneath the orifice, tube, or cock; from this small trough the water ran into one or more circular brass vessels L, which were weighed when the experiment was over, on a scale Z, reading to half ounces.

The time was furnished by a stop-watch, giving quarter seconds, and the diameters of orifices, tubes, &c., &c., were determined by means of tapering sheet-metal gauges and solid conical brass rods, measured with Brown and Sharp's Vernier calipers, reading to 0·001 inch.

When the discharge took place under water the cylindrical brass reservoir A, was connected with a square brass box H, 3 inches wide by 2 inches high in the clear, and some 16 inches long, resting on the upper parallel plate B, by means of one or more brass tubes W, nearly $2\frac{3}{4}$ inches in diameter and $2\frac{1}{4}$ inches high, screwed together, the connections being rendered perfectly watertight by the interposition of rubber bands between the brass bearing surfaces.

On top of the square brass receiving box H, and near one end thereof, stood a glass graduated tube N, open at both ends, of inch bore, some 50 inches high, hermetically connected with the square box by means of a stuffing box; this tube served the double purpose of indicating approximately the height of the water or intensity of the pressure in the receiving reservoir and preventing any accumulation of air therein.

The water that passed from the upper cylindrical reservoir A, through a submerged orifice or tube fitted into its bottom, was discharged through a $\frac{3}{4}$-inch gauged cock V, inserted in a stuffing box at the left end of the square receiving reservoir H, into the light conduit or trough of sheet copper K, already referred to, whence it ran finally into the brass vessel L, until the time allotted for each experiment, viz., usually from 100 to 300 seconds, was up, when the trough was quickly removed from under the cock V, and the water allowed to go to waste; everything, in other respects, remaining undisturbed, until it was settled whether or not it was desirable to repeat the experiment.

The square box or receiving reservoir H, was connected at the right end by means of an India rubber tube P, $\frac{3}{4}$ inch diameter inside, provided with brass couplings, with a cylindrical vessel Q of sheet copper, 6 to 8 inches in diameter, and some 3 inches high, supported on a movable bracket pushed tightly into one of the interstices, 1 inch high, left between every two of a tier of shelves let into two uprights,

* The tank had an area of 36 feet, and was supplied from the water works of the City of Ottawa by means of an inch service pipe, provided with a bib and ball cock, and its water surface stood, on an average, say, 16 feet above the water in the reservoir.

raised on a heavy base, the whole of wood, so as to form a firm stand R; by this means the water surface in the receiving reservoir Q, could be fixed at any elevation below that of the reservoir of supply A, that might be found desirable. A second hook-gauge, with scale S_2, and vernier, supported on a bracket similar to that just described, which was inserted into a compartment situated at a convenient height above the top of the reservoir Q, served to determine the actual difference of level between the water surface of this reservoir and that of the reservoir of supply A, to within $\frac{1}{500}$ part of an inch.

Prior to commencing a set of experiments, the zero points of the scales S_1 and S_2, in connection with the respective reservoirs A and Q, were compared with each other, by taking the elevation of the water surface in both of them, while the liquid was in a state of perfect equilibrium in the whole system of vessels and tubes, proper care being taken that no leakage or syphoning should take place anywhere, and sufficient time allowed for the water to come to a perfect stand still in each case.

When it was found requisite to use a greater head of water than that which could be directly furnished by the cylindrical reservoir A, viz., about 3 inches, the orifice plates or tubes experimented with were screwed into the bottom of an auxiliary brass cylinder U, some 3 inches in diameter inside, and 8 inches high. This auxiliary cylinder U, itself, was then screwed into the bottom of the 12-inch reservoir A, in the place of the hook-gauge stand G, and placed in communication with the iron 1 inch supply pipe, from the tank in the garret, by an intermediate $\frac{3}{4}$-inch rubber hose. The effective pressure on the orifice or tube was regulated by the inlet cock, its intensity being ascertained by observing to what height the water rose in a glass tube connected with the 3-inch closed reservoir, at its highest point, by means of a flexible rubber tube X.

EXPERIMENTS.

COEFFICIENTS OF DISCHARGE THROUGH CIRCULAR ORIFICES, IN THIN PLATES.

It is generally conceded by all authorities in hydraulic matters, such as Michelotti, Bossut, Eytelwein, Venturi, D'Aubuisson, Weisbach, &c., that, for a circular orifice in a thin plate, the coefficient of velocity of efflux, corresponding to the plane of the orifice—that is to say, the ratio between the quantity of water actually discharged and the quantity which would be discharged from the reservoir if the velocity in the plane of this orifice was equal to that acquired by a heavy body falling freely or in vacuo, through a space equal to the height of the water surface, above the centre of the orifice—varies between 0·60, or thereabouts, for large heads and small circular orifices, and 0·66 or 0·68 for small heads and large orifices, when the discharge takes place in the open atmosphere.

I may remark, however, at the outset, that the experiments with small orifices, under large heads, on record, are not very numerous, so far as I have been able to find out, and to say the least, those that are available do not inspire unlimited confidence as to the accuracy of the results arrived at. Thus—while Michelotti found the coefficient of velocity of efflux to be 0 607, for an orifice 2·126 inches in diameter, under a head of 7·218 feet, and 0·597 for a circular orifice, the diameter of which was 3·189 inches, under a head of 22.179 feet—Weisbach says that for an orifice of 1 centimeter, or about 0·394 inch in diameter, this coefficient is: $0{\cdot}632 \times 0{\cdot}99 = 0{\cdot}6256$, under a head of 13·574 meters, or 44·536 feet, and $0{\cdot}60 \times 0{\cdot}994 = 0{\cdot}5964$, under a head of 103,578 meters, or 339,839 feet; these last two co-efficients appear to me to be much too large, or else the two former are too small.

The coefficients of velocity determined by myself for efflux, in air, through circular orifices in a thin plate, do not differ from those obtained by a number of others, before me, under similar circumstances, as may be seen by the following recapitulation of experiments, headed Table 1.

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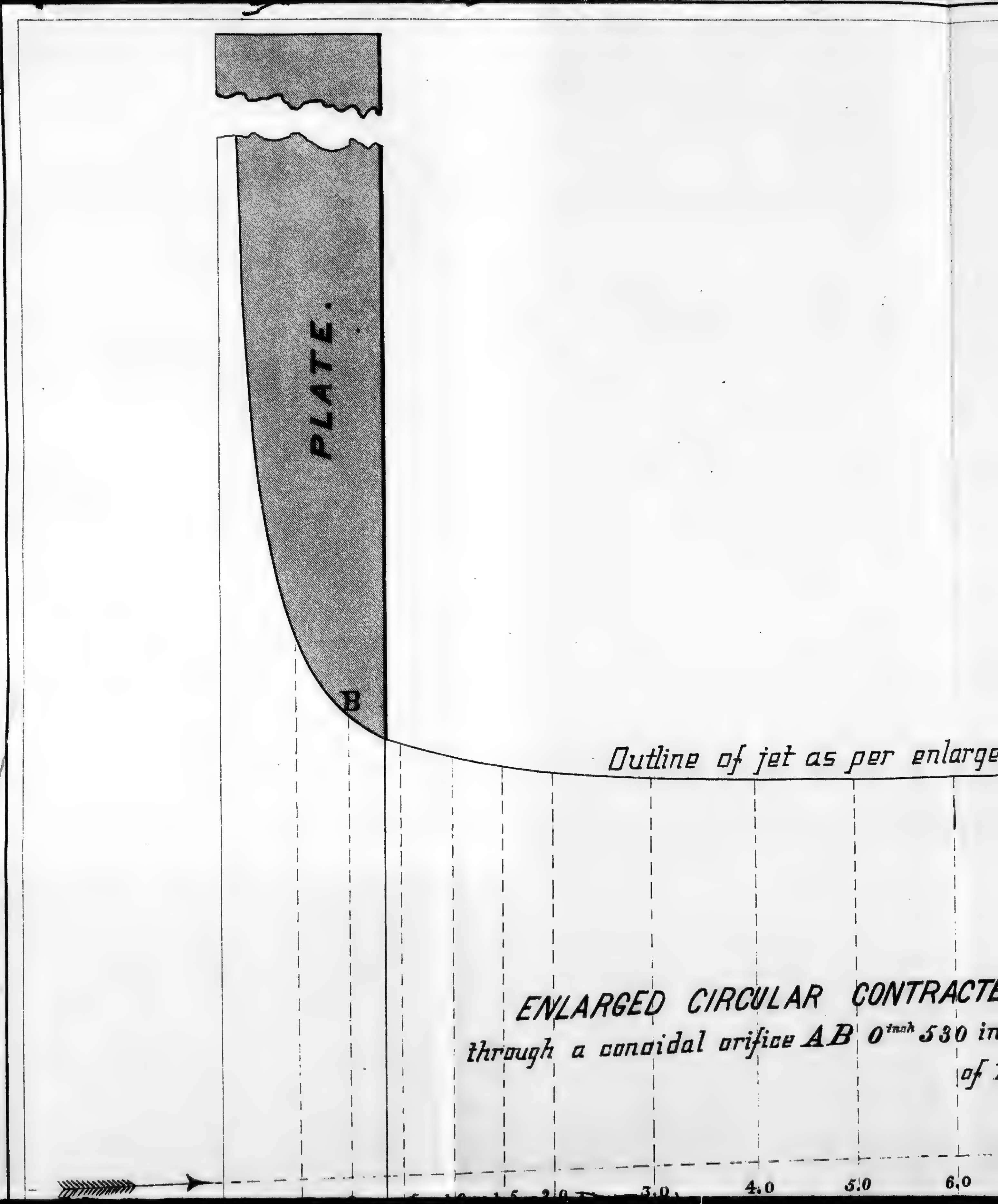
PLATE.
B
Outline of jet as per enlarge
ENLARGED CIRCULAR CONTRACTE
through a conoidal orifice AB 0 inch 530 in
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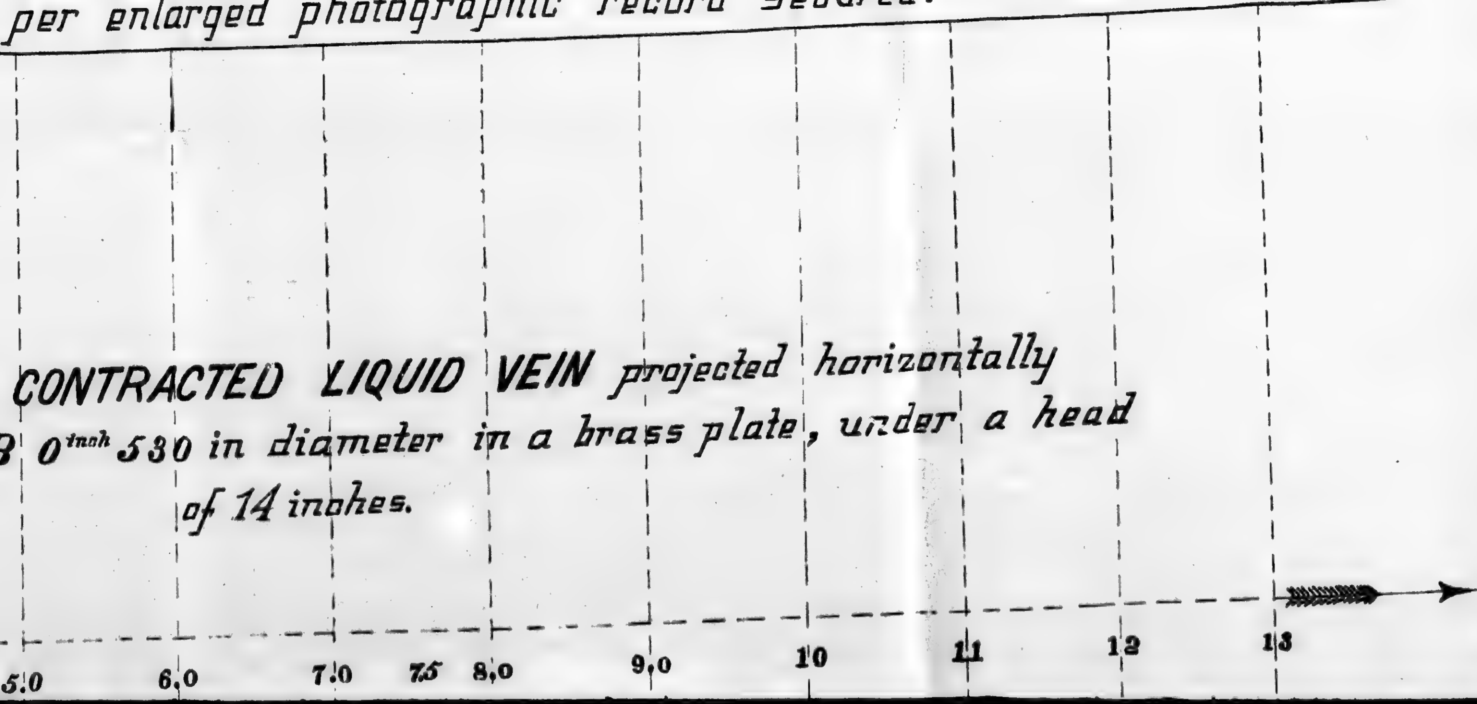
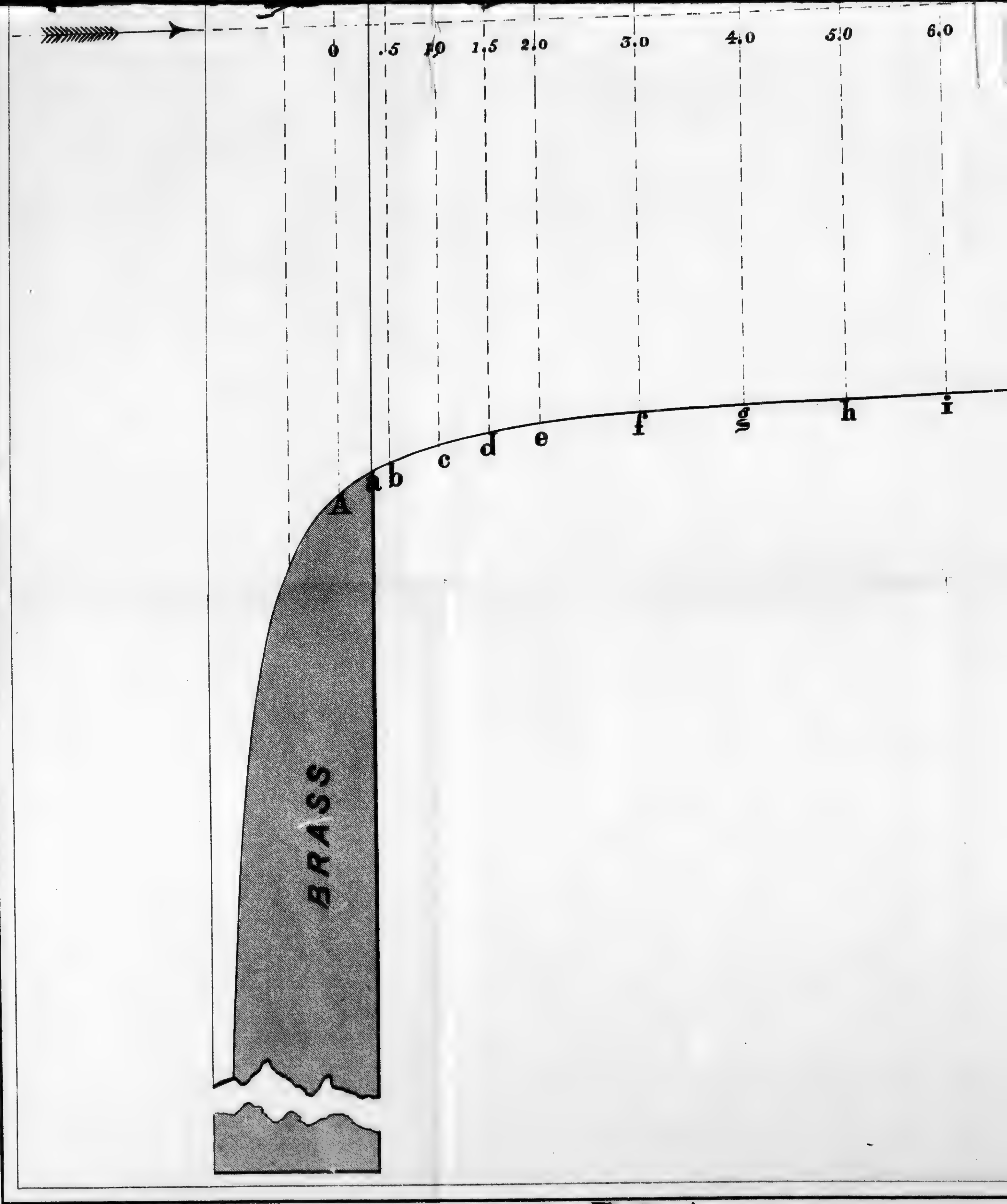
PL. II.

per enlarged photographic record secured.



CONTRACTED LIQUID VEIN projected horizontally
3 0inch 530 in diameter in a brass plate, under a head
of 14 inches.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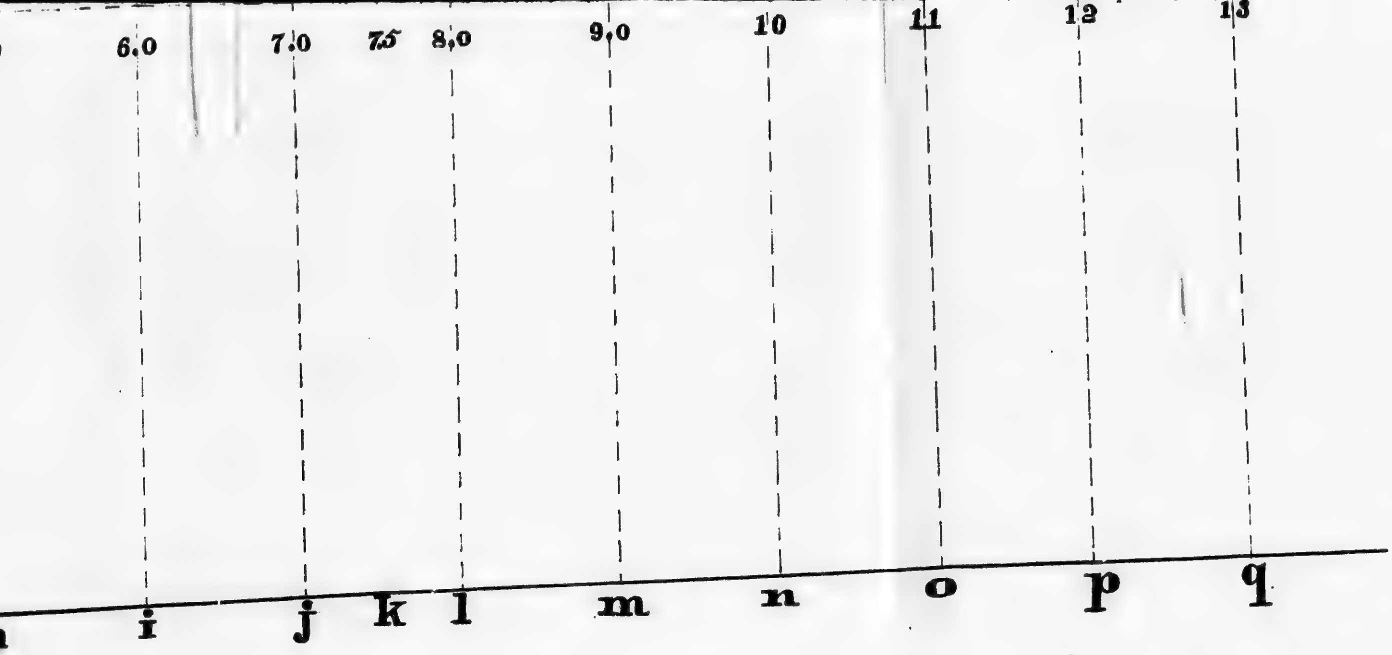
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Letter of reference.	Num... exper... m...
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TABLE I.

Letter of reference.	Number of experiments made.	Diameter of orifice in inches.	Mean head in inches.	C $\left(\frac{\text{vel.}}{\text{orif.}}\right)$ Average value of coefficint of velocity of efflux, in air, at plane of orifice. (1)	Remarks.
A	3	0·384	51	0·6210	The diameter of each orifice was obtained by measuring, with Brown & Sharpe's vernier calipers, reading to 0·001 inch, a slightly conical brass mandrel introduced into the hole, at the point where it filled the same, the largest dimensions being assumed to be nearest the true one.
B	3	"	44	0·6263	$\sqrt{2g}$ was taken at 27·78 in inches; 1 ounce was taken equal to 1·7315 cubbic inches.
C	2	"	35	0·6259	
D	"	"	29	0.6277	
E	"	"	19	0·6268	
F	"	"	12·10	0·6281	
G	8	"	3·08	0·6544	
H	6	0·400	2·97	0·6703	
I	6	"	2·92	0·6727	
J	5	0·4185	3·03	0·6802	
K	14	0·420	3·07	0·6775	
L	3	0·482	3·00	0·6803	
M	4	0·484	2·81	0·6844	

In order to establish coefficients of efflux for very small heads and large orifices, I made experiments with submerged orifices. A synopsis of the results arrived at is given in—

TABLE II.

Letter of reference.	Number of experiments made.	Diameter of orifice in inches.	Mean head in inches.	$C\left(\frac{\text{vel.}}{\text{orif.}}\right)$ Average value of coefficient of velocity of efflux, under water, at plane of orifice.	Remarks.
A	7	0·484	0·12	0·6615	
B	7	"	0·13	0·6564	
C	4	"	0·23	0·6540	
D	3	"	0·38	0·6531	Temperature of water, from 52° to 55° Fahrenheit.
E	7	"	0·50	0·6528	
F	3	"	1·42	0·6532	
G	2	"	2·60	0·6503	
H	10	1·031	0·040	0·6598	
I	"	"	0·053	0·6684	
J	"	"	0·103	0·6676	
K	"	"	0·155	0·6619	
L	"	"	0·206	0·6639	

On comparing the above coefficients for discharge under water, with corresponding ones for efflux in air, given in Table I, it is found that from 4½ to 5 per cent must be subtracted from the coefficients of efflux in air, to convert them into co efficients of efflux under water, instead of only 1⅓ per cent. obtained by Dr. Weisbach for ordinary heads of water I suppose, * indicating a difference of over 3 per cent., which, although comparatively large, may still properly be considered to be due, in a great measure, to the very small heads which I used exclusively.

The coefficients to be used for efflux under water through circular orifice in thin plates, which are given by Mr. J. B. Francis, in his "Lowell Experiments," differ very materially from those obtained by myself, as recorded above, in Table II., and still more from those established according to Dr. Weisbach's rule, just referred to (1).

Mr. Francis entertains, apparently, no doubt but that the coefficient of efflux, through a circular submerged orifice, 0·1017 foot = 1·2204 inch in diameter, should not exceed 0·57 under small heads of from 1 to 5 inches, for at page 225 of his work (1), he says: "It is the general result of the great number of experiments, on record "on the flow of water through orifices in a thin plate, discharging freely into air, that "the coefficient of discharge (which in simple orifices is the same thing as the ratio "of the velocity at the smallest section of the orifice to the velocity due to the head) "is greatest for very small heads. In these results where the discharge takes place

* See Weisbach's Mechanics of Engineering and of the Construction of Machines.—English Translation, by Cox, page 825.

(1). See "Lowell Hydraulic Experiments by J. B. Francis."—Third edition, 1871—D. Van Nostrand, N.Y.—Table XXVII—Experiments 93 to 101.

"under water, the coefficient of discharge is least with the very small heads. This "result is so marked and uniform that there can be no doubt of the fact."

Nevertheless, my fifty experiments, H, I, J, K, L, Table II, indicate unmistakeably that even under the very small heads, varying between $\frac{1}{10}$ and $\frac{1}{20}$ of an inch, the coefficient in question is at least as high as 0 66, for an orifice of 1.031 inch in diameter.

The only distinctive feature that I can see in Mr. Francis' experiments on submerged circular orifices in a thin plate, as compared to my own, is that his orifice, of 1·22 inch, was in a vertical plane, while my orifice of 1·031 inch in diametr was in a plane parallel to the horizon.

I may be allowed to observe, in regard to the discrepancies found to exist between Mr. Francis' coefficients and those of other experimenters, for efflux under very small heads, that his mode of establishing the quantity of liquid flowing in a given time, through a circular orifice in a thin plate, 1·2204 inch in diameter under small heads, varying from say 1 to 5 inches by means of the measured depths of the contracted stream passing over the sharp crest of a weir 7·8 inches long, placed in the wall at the far end of a rectangular reservoir 11¼ feet long and 3·0 feet wide, viz.: 6¼ feet beyond the plane of the discharging orifice, does not appear to me to be one calculated to lead to unquestionable results.

I do not see that it is possible to determine, with unerring certainty, the discharging power of an orifice in a reservoir, otherwise than by weighing the quantity of water which actually flows out of it into a receiving vessel in a given time and under a constant head, and I consider this to be more especially the case when the heads used are small and the reservoirs comparatively large. I cannot help thinking that had Mr. Francis bored small holes in the wall wherein the weir was placed, at a depth of 1 foot or so below the level of the crest, and weighed the water that would have flowed in a fixed space of time, out of the openings, taking one after another or as many together as would have been convenient, he would very probably have arrived at a different conclusion respecting the value of the coefficients of efflux which are applicable when submerged circular orifices, in thin plates, are used.

On the whole, I think we can admit with confidence that the coefficient of efflux, in air, through my orifice of 1·031 inch in diameter, would be, under the very small head of about $\frac{1}{20}$ inch—if such a vein could be produced in its complete state—in air, no less than 0·668 + 0·032 additional for discharge in air, instead of water, viz.: 0·70; even this value is perhaps yet slightly smaller than it would be if ordinary river water was a perfect fluid in every respect.

The Chevalier Lorgna contends that the reduced velocity of the liquid in the plane of the orifice, as compared with the ordinary theoretical velocity; $V=\sqrt{2gH}$ due to the head, H, of water in the reservoir, is due to the simultaneous pressure of the whole liquid mass around the orifice, which, he says, prevents the free efflux from the reservoir; and he computes the theoretical velocity in the plane of the orifice to be:

$$V_{\text{orif.}} = \left(\sqrt{2\left(\frac{\sqrt{5}-1}{2}\right)^3}\right)\sqrt{2gH} = \sqrt{.472127\times 2gH} = 0{\cdot}687115\cdot\sqrt{2gH}.$$

Mr. H. Resal proves (see article 268, page 288, second volume of his "Traite de Mecanique Générale"—Paris—Gauthier Villars, 1874) that the coefficient of discharge through an orifice, in a thin plate, can never be less than ½ or 0 5.

COEFFICIENTS' OF CONTRACTION.

It has been usual to take for granted that the coefficient of contraction of the circular vein projected from an orifice in a thin plate, becomes a minimum at a distance from the orifice, equal, on an average, to once or twice its radius. At or near this point, the diameter of the vein has been measured repeatedly by means of four pointed set screws, mounted on a circular diaphragm, these screws being directed, by the eye, as nearly as possible, towards the centre of the vein, until the points touched

its surface. The mean of the two distances, between opposite points, has been invariably held to be the true diameter of the vein at its greatest contraction; this diameter was found to be, on an average, 0·8 of that of the orifice.

From the manner just described, in which these coefficients of contraction are commonly obtained, it is manifest that although they are, as a general thing, sufficiently accurate for practical purposes, for the objects of theoretical research they are not equally serviceable.

In order to arrive at something more reliable, in my opinion, I measured two vertically descending veins, projected through circular orifices, in thin plates of 0·4 inch and 0·482 inch in diameter, respectively, under a constant pressure of some 3 inches.

For this purpose, the position of the cylindrical reservoir of supply A (See fig. 2), into the bottom of which the orifice plates were screwed, was adjusted by means of the four levelling screws D, so as to render the plane of the orifice truly horizontal in every case. The diameter of the vein was measured at various points by means of pointed screws, mounted opposite each other on a circular diaphragm *d*, secured with a screw *c*, to a vertical cylindrical brass standard *r*, along which it could be moved up or down, by sliding. The foot of this upright brass rod *r*, was ground to fit closely into each of three long vertical tapering sockets *s*, united by three radiating bars to a central ring, so as to form a kind of tripod, which was placed concentrically under the falling liquid vein.

The rod *r*, together with the diaphragm *d*, was turned round in each of these sockets, until I succeeded in adjusting the positions of the screws, so that their points would describe, about the centre or axis of the rod or socket, circular arcs tangent to the liquid vein at both sides. The distance between the points of the screws was then ascertained, by measuring, at the proper place, with the vernier callipers, already described, the diameter of a conical mandril introduced between them.

The dimensions and coefficients of contraction found are given in Tables III and IV, which here follow :—

TABLE III.

Liquid Contracted Vein, falling vertically under a head of 2·99 inches, through a circular orifice, in a thin horizontal plate, 0·4 inch in diameter.

Letter for reference.	x, Abcissa, or distance from the plane of the orifice, down to the measured section.	$2y = d$. Diameter of the vein.	h. Depth of the measured section below the water surface.	$C_{\text{cont.}} = \left\{ \dfrac{\sqrt[4]{h}}{\sqrt[4]{2 \cdot 99}} \right\} \dfrac{d}{\cdot 4}$ Coefficient of contraction, abstraction being made of the acceleration produced by gravity, outside of the reservoir.* see foot note next page)
	Inches.	Inches.	Inches.	
A	0·000	0·400	2·990	1·0000
B	0·800	0·309	3·790	0·8197
C	1·000	0·303	3·990	0·8143
D	1·535	0·296	4·525	0·8207
E	2·535	0·282	5·525	0·82 0
F	3 535	0·270	6·525	0·8203
G	4·535	0·258	7·525	0·8063
H	5·535	0·248	8·525	0·8056
I	6·535	0 242	9·525	0·8083
J	7·535	0·238	10·525	0·8116
K	8·035	0·234	11·025	0·8082
L	8·535	0·231	11·525	0·8089
M	8·800	0 229	11·790	0·8070
N	9·535	0·227	12·525	0·8118
O	10·535	0·224	13·525	0·8165
P	11 535	0·220	14·525	0·8165
Q	12 535	0·216	15·525	0·8150
R	13·535	0·212	16·525	0·8127
S	14·535	0·209	17 525	0·8129
T	15·535	0·207	18·525	0·8165
U	16·930	0·205	19 920	0·8086

Mean value of C_{cont} = say 0·813, whence $C^2_{\text{cont}} = 0{\cdot}813^2$ 0·661.

Coefficient of velocity of efflux, $C_{\left(\text{vel. orif.}\right)} = 0{\cdot}6662$, whence $C^2_{\left(\text{vel. orif.}\right)} = 0{\cdot}44382$.

Coefficient of velocity at section of greatest contraction $= \dfrac{0{\cdot}6662}{0{\cdot}6610} = 1{\cdot}0078$.

TABLE IV.

LIQUID Contracted Vein, falling vertically under a head of 3·00 inches, through a circular orifice, in a thin horizontal plate, 0·482 inches in diameter.

Letter for reference.	x, Abcissa, or distance from the plane of the orifice, down to the measured section.	$2y = d.$ Diameter of the vein.	$h.$ Depth of the measured section below the water surface.	$C_{\text{cont.}} = \left\{\frac{\sqrt[4]{h}}{\sqrt[4]{3\cdot00}}\right\} \frac{d.}{\cdot482}$ Coefficient of contraction, abstraction being made of the acceleration, produced by gravity outside of the reservoir.*
	Inches.	Inches.	Inches.	
A	0·000	0·482	3·000	1·0000
B	0·925	0·380	3·925	0·8431
C	1·925	0 358	4·925	0·8407
D	2 925	0·341	5·925	0·8387
E	3·925	0 327	6·925	0·8366
F	4 925	0·316	7·925	0·8353
G	5·925	0·306	8·925	0·8337
H	7·535	0·289	10·535	0·8205
I	10·535	0·279	13·535	0·8436
J	13·535	0·260	16·535	0·8263

* At a distance of one to two diameters below the plane of the oriffce, the vein-form is here supposed to be governed only by the ordinary laws of the descent of heavy bodies subjected to the forec of gravity.

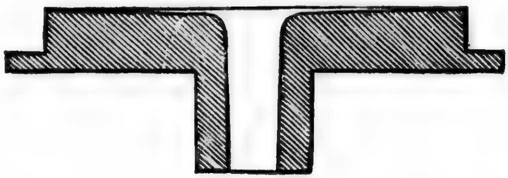
Mean value of $C_{\text{cont.}}$ = say 0·835, whence $C^2_{\text{cont.}} = 0\ 835^2 = 0\cdot6972$

Coefficient of velocity of efflux $C_{\left(\substack{\text{vel.}\\\text{orif.}}\right)} = 0\cdot6803$, whence $C^2_{\left(\substack{\text{vel.}\\\text{orif.}}\right)} = 0\cdot46281.$

Coefficient of velocity at section of greatest contraction $= \frac{0\cdot6803}{0\cdot6972} = 0\cdot9758.$

In order to gain, at least, an approximate knowledge of the rate of variation of the coefficients of contraction applicable to liquid veins in general, I made the experiments under various heads, which are recapitulated in Table V, with a polished brass mouth-piece, having nearly the form of the contracted vein projected through a circular orifice in a thin plate, of 0·4 inch diameter, under a head of say between 1 and 2 feet.

Fig. $2\frac{1}{2}$



This mouth-piece or artificial contracted vein, shown full size in Fig $2\frac{1}{2}$, is 1 inch long, the diameter of the bore at the small end being 0·313 inch, while at the junction with the reservoir its cross-section may be considered to be infinitely great as compared to that of the small end.

The coefficients of contraction, $C_{\text{cont.}}$ given below in Table V, were computed on the supposition that inasmuch as the form of the mouth-piece coincided nearly with the true conoïdal form which the naturally contracted vein would assume, in each case, the fluctuations of the coefficients of discharge, $C_{\text{disch.}}$ were entirely due to defficiency of the waterway afforded by the mouth-piece in comparison to the areas of the respective corresponding cross-sections of the natural contracted veins projected under equal heads through an orifice of 0·4 inch in diameter.

As the actual amount of acceleration produced by gravity during the passage of the liquid downward, from the large to the small base of the mouth-piece, in addition to that due to the hydrostatic pressure in the reservoir, cannot be computed with unerring certainty, when the efflux takes place in air, I preferred to have the discharge take place under water, running the risk of having to apply, for efflux in air, approximate corrections to the coefficients as found for discharge under water.

TABLE V.

1	2	3	4	5	6	7	8		9	10	11	12
No. of Experiments.	Elevation of water in reservoir of supply A, above 0 of hook-gauge scale, or 0 of glass tube.	Elevation of water in receiving reservoir above 0 of hook-gauge scale, or 0 of glass tube.	Difference of level between the surfaces of the two reservoirs, or effective head.	h Mean effective head.	T Duration of experiments.	Designation of vessels.	Total weight of the vessels, with the water contained therein, at the end of each experiment.		D Total mean net discharge.	d Discharge per second $= \frac{1\cdot7315D}{T}$	V Velocity per second at small base of mouthpiece, $= \frac{d}{a}$, a representing the area of this base.	$\sqrt{2gh} = 27,78\sqrt{h}$ Theoretical velocity due to the mean effective head h.
	inches.	inches.	inches.	inches.	sec.		lbs.	oz.	ounces.	cub. in.	inches.	inches.
1	66·000	8·000	58·000	58·000	50	V_o	34	5	459	15·8951	206·4308	211·638
2	66·000	8·000	58 000		50	V_o	34	5				
3	58·600	8 000	50·600	50·600	50	V_o	32	5	427	14 7870	192·0390	197·5991
4	58 600	8 000	50·600		50	V_o	32	5				
5	51·600	8 000	43·600	43·600	50	V_o	30	5	395	13 8788	177·6474	183·4321
6	51·600	8 000	43·600		50	V_o	30	5				
7	43·800	8·000	35 800	35·800	50	V_o	27	14	356	12 3283	160·1076	166 2163
8	43·800	8·000	35·800		50	V_o	27	14				
9	38·000	8 000	30·000	30·000	50	V_o	26	0	326½	11·3067	146·8402	152·1511
10	38·000	8·000	30·000		50	V_o	26	1				
11	32·400	8·000	24·400	24·400	100	V_o	42	4	586½	10 1553	131·8863	137·2230
12	32·400	8 000	24·400		100	V_o	42	5				
13	24·200	8·000	16 200	16·200	100	V_o	35	4½	475	8·2246	·106·8126	111·8145
14	24·200	8·000	16·200		100	V_o	35	5½				
15	19·700	8·000	11·700	11·700	100	V_o	30	9	390	6·9087	89·7232	95·0222
16	19·700	8·000	11·700		100	V_o	30	9				
17	+ 3·114	— 2·688	5·800	5·800	200	V_o	40	10¼	560½	4 8525	63·0198	66·9031
18	+ 3·078	— 2·154	5 232	5·233	200	V_o	38	14½	532¼	4 6079	59 8436	63·5488
19	+ 3 078	— 2 156	5·234		200	V_o	38	14				
20	+ 3·080	— 1·230	4·310	4·310	200	V_o	35	10½	480½	4·1600	54·0250	57·6727
21	+ 3·082	— 0·732	3·814	3 814	200	V_o	33	13	451	3·9045	50·7082	54·2543
22	+ 3·074	— 0·540	3·614	3·611	300	V_o	46	4	651	3·7573	48·7968	52·7895
23	+ 3·072	— 0·536	3 608		300	V_o	46	6				
24	+ 3·110	+ 0·100	3·010	3·007	300	V_o	41	8	589½	3·4024	44·1870	48·1723
25	+ 3·104	+ 0·100	3 004		300	V_o	42	7				
26	+ 3·066	+ 0·620	2·446	2,439	300	V_o	38	14	532	3·0705	39·8770	43 2851
27	+ 3·084	+ 0·652	2·431		300	V_o	38	14				
28	+ 3·088	+ 1·220	1·868	1·875	300	V_o	33	13½	451¾	2·60735	33·8617	38·0393
29	+ 3·092	+ 1·210	1·882		300	V_o	33	14				
30	+ 3·090	+ 1 964	1·126	1·112	300	V_o	26	13	339	1·9566	25·4103	29·2945
31	+ 3·088	+ 1 990	1·098		300	V_o	26	13				
32	+ 3·068	+ 2·500	0·568	0·573	300	V_o	19	6	220	1·2697	16·4905	21·0285
33	+ 3·082	+ 2·496	0·586		300	V_o	19	6				
34	+ 3·080	+ 2·516	0·564		300	V_o	19	6				
35	+ 3·072	+ 2·874	0·198	0·186	300	V_o	10	5	109·5	0·6320	8·2078	11·9815
36	+ 3·072	+ 2·900	0·172		300	V_o	10	5				
37	+ 3·064	+ 2·878	0·186		300	V_o	10	4½				
				0·000			0	0	0	0	0	0

TABLE V.

11	12	13	14	15	16	17	
Velocity per second at small base of mouthpiece, $=\frac{d}{a}$, a representing the area of this base.	$\sqrt{2gh} = 27{,}78\sqrt{h}$ Theoretical velocity due to the mean effective head h.	$C_{disch} = \frac{v}{\sqrt{2gh}}$ Coefficient of discharge under water	$m = \cdot06 \cdot \frac{\sqrt{3\cdot13}}{\sqrt{3\cdot13+h}}$ Correction to be added to the coefficient of discharge, C_{disch} for efflux under water, to reduce it to the co-efficient of discharge for efflux in open air, viz., to C'_{disch}	$C'_{disch} = C_{disch} + m$ Coefficient of discharge in the open air	$C_{cont} = \cdot813\sqrt{\frac{0\cdot9556}{C_{disch}}}$ Coefficient of contraction, based on the co-efficient of contraction obtained by direct measurement of the descending veins, projected through an orifice, in a thin plate 0·4 in. diam., viz., ·813 in experiment 25.	$C^4_{cont} = \frac{r^4_{cont.}}{r^4_{orif.}}$	Remarks.
inches.	inches.						
206·4308	211·6380	0·9751	0·0132	0·9883	0·79914	0·40845	
192·0390	197·5991	0·9718	0 0137	0 9855	0·80057	0 41077	
177·6474	183·4321	0 9684	0·0147	0·9831	0·80155	0·41278	
160·1076	166 2163	0 9632	0 0160	0·9792	0·80314	0·41607	
146·8402	152·1511	0·9651	0 0174	0·9825	0·80180	0 41328	
131·8863	137·2230	0·9611	0·0189	0 9800	0 80281	0·41540	
·106·8126	111·8145	0 9552	0·0225	0 9777	0·80376	0·41735	
89·7232	95·0222	0·9442	0·0254	0·9696	0·80711	0 42435	
63·0198	66·9031	0·9141	0 0327	0·9768	0·80418	0·41812	
59 8436	63·5488	0·9417	0·0337	0·9754	0 80470	0·41932	
54·0250	57·6727	0 9368	0·0355	0·9723	0 80599	0·42200	
50·7082	54·2543	0·9346	0·0367	0 9713	0·80640	0 42287	
48·7968	52·7895	0·9244	0·0368	0·9612	0 81063	0 43180	
44·1870	48·1723	0 9173	0·0383	0·9556	0·81300	0·43688	
39·8770	43 2851	0·9191	0·0403	0·9594	0·81139	0·43343	
33·8617	38·0393	0·8902	0 0412	0·9314	0·82349	0 45988	It is probable that, owing to the very small heads used in experiments No 28 to 87, the coefficients of discharge and contraction are sensibly affected by friction.
25·4103	29·2945	0·86741	0 0436	0·9110	0·83266	0·48070	
16·4905	21·0285	0·78419	0·0422	0 8264	0 87424	0·58416	
8·2078	11·9815	0·6850	0 0389	0·7239	0·93409	0·76130	
0	0	0·6647 supposed limiting value.	0·0424 supposed limiting value.	0 7071 supposed limiting value	1·0000 supposed limiting value.	1·0000 s'pposed limiting value.	

The following are the results obtained by Michelotti, the younger, with large jets, under great heads. He refers the curve assumed by the longitudinal profile of the contracted vein to a cycloid, and in one of his experiments with a cycloidal tube, he found the coefficient of velocity at the section of maximum contraction to be 0·981:

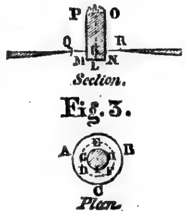
TABLE VI.

h Head above the orifice in feet.	Diameter in inches.		$C_{cont.}$ Coefficient of contraction or ratio between the diameters.	Distance from orifice to contraction, in inches.	Ratio of the distance to the contracted diameter.	$C^4_{cont.}$
	At the orifice.	At the contraction.				
6·890	6·394	5 047	0 790	2·520	0·501	0 3895
12·008	6·394	5·039	0·788	2·520	0·500	0·3856
7·349	3·197	2·511	0·786	1·260	0·500	0·3817
12·502	3·197	2·504	0·783	1·210	0·492	0·3759
22·179	3·197	2·413	0·755	1·181	0·497	0 3249

Mr. H. Résal says, at page 290, vol. ii., of his "Traité de mécanique générale" (Paris, Gauthier Villars, 1874), that the results of experiment respecting the contraction of a liquid vein through a circular orifice in a thin plate, show that for any head less than 6·80 met. = 22·3088 feet = 267·7038 inches, the co-efficient of contraction is equal to $\sqrt{\cdot 62}$, or ·7874—for all orifices the diameter of which is less than $0^{m}16$ = 6 299 inches, and greater than $0^{m}02$ = ·78737 inches.

EXPERIMENTS ON THE FLOW OF LIQUID THROUGH ANNULAR SPACES FORMED BY INTRODUCING A CYLINDRICAL ROD OR DISK, INTO A CIRCULAR ORIFICE, PIERCED IN A THIN PLATE.

EXPERIMENTS ON THE FLOW OF LIQUID THROUGH ANNULAR SPACES FORMED BY INTRO- DUCING



Section.

Fig. 3.

Plan.

I. The discharge took place, in air, under a uniform head, throug
an orifice in a thin plate, 0·4 inch diameter, and the surface G H I
axis, J K L, through the centre of the orifice, to points at variou
Area, a, of the annular opening, A B C I G H, = 0·09980 square

$$\frac{\text{Area A B C G H I}}{\text{Area A B C}} = \frac{0 \cdot 098800}{0 \cdot 125664} = 0 \cdot 78622.$$

Ratio of breadth, A G, of ring-shaped opening to its mean length

TABLE VII.

1		2	3	4	5	6	7	8		9	10	11
Series and Nos. of experiments.		Elevation of water surface, in reservoir of supply, A, above 0 of hook-gauge scale.	Mean elevation of water-surface in reservoir of supply.	Elevation of the plane of the orifice in a thin plate, A B C, referred to 0 of the hook-gauge scale.	h Mean head of water on the horizontal orifice A B C.	T Duration of experiments.	Designation of vessels.	Total weight of the vessels with the water contained therein, at the end of each experiment.		D Total mean net discharge.	d Discharge per second in cubic inches $=\frac{1 \cdot 7315\ D}{T}$	Velocity $\frac{d}{a}$ = per second.
		inches.	inches.	inches.	inches.	seconds		lbs.	oz.	ounces.	cub. inchs.	inches
	1	3·980		1·016		100	V_1	15	13			
a	2	3·936	3·982	1·016	2·966	,,	V_1	,,	13	197·5	3·4197	34·612
	3	3·980		,,		,,	V_1	,,	13			
	4	3·960		,,		,,	V_1	,,	5			
b	5	3·958	3·958	,,	2·942	,,	V_1	,,	5½	189·5	3·2812	33·210
	6	3·958		,,		,,	V_1	,,	5			
	7	3·996	3·996	,,	2·980	,,	V_1	,,	3	187·5	3 2466	32·860
	8	3·980	3·980	,,	2 964	,,	V_1	,,	2¼	187·0	3·2379	32·772
c	9	3·970	3·970	,,	2·954	,,	V_1	,,	2¼	186·75	3·2335	32·804
	10	3·932	3·932	,,	2·916	,,	V_1	,,	1¾	186·0	3·2206	32·597
	11	3·904	3·904	,,	2·888	,,	V_1	,,	¾	185·0	3·2033	32·421
	12	3·982	3·982	,,	2·966	,,	V_1	,,	1	185·5	3·2118	32·509
d	13	3 982	3·982	,,	2·966	,,	V_1	,,	1	185·5	3·2119	32.509
	14	3 950	3·950	,,	2·934	,,	V_1	,,	¾	185·0	3·2032	32·421
	15	3·980	3·980	,,	2·964	,,	V_1	,,	14¾	199·0	3·4457	34·875
e	16	3·980	3·980	,,	2·964	,,	V_1	,,	14¾	199·0	3·4457	34·875
	17	3 980	3·980	,,	2·964	,,	V_1	,,	14¾	199 0	3·4457	34·875
	18	3·982		,,		,,	V_1	16	3¼	....	....	....
f	19	3·980	3·980	,,	2·964	,,	V_1	,,	3½	204·0	....	....
	20	3·980		,,		,,	V_1	,,	3	....	....	....
	21	3·970		,,		,,	V_1	,,	11½	....	....	....
g	22	3·970	3 970	,,	2·954	,,	V_1	,,	12	212·5	....	....
	23	3·970		,,		,,	V_1	,,	12	....	....	....
	24	3·966		,,		,,	V_1	17	6½	....	....	....
h	25	3·966	3·966	,,	2·950	,,	V_1	,,	6	222·5	....	....
	26	3·964		,,		,,	V_1	,,	6	....	....	....
	27	3·954		,,		,,	V_1	,,	14½	....	....	....
i	28	3·954	3·954	,,	2·938	,,	V_1	,,	14	230·5	....	....
	29	3·954		,,		,,	V_1	,,	14	....	....	....
	30	3·954		,,		,,	V_1	18	0	....	....	....
j	31	3·956	3·956	,,	2·940	,,	V_1	,,	0	232·5	4·0257	32·035
	32	3·956		,,		,,	V_1	,,	¼	....	....	....
	33	3·956	3·956	,,	2.940	,,	V_1	,,	0	232 5	4·0257	32·035

DUCING A CYLINDRICAL ROD OR DISK, INTO A CIRCULAR ORIFICE, PIERCED IN A THIN PLATE.

through the horizontal annular space left between the the circumference, A B C, of G H I of a cylindrical rod, M N O P, 0· 185 inch in diameter, let down along the various disiances, K L, above and below the plane of the said orifice.

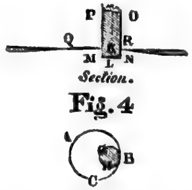
square inch. Area of orifice A B C = 0·125664 square inch. Hence

length D E F, measured in the centre $= \frac{0·918918}{0·107500} = 8·55.$

VII.

11	12	13	14	
Velocity per second. $v = \frac{d}{a}$	$\sqrt{2gh} = 27·78 \sqrt{h}$.	Coefficient of velocity or discharge. $C_{disch.} = \frac{v}{\sqrt{2gh}}$	Distance, K L, between the base M N, of the cylindrical rod and the plane of the orifice, + above it, — below it.	REMARKS.
inches.	inches.		inches.	
			0·000	The brass vessel, Vt, weighed 55·5 ounces.
34·6125	47 8371	0·7256	0·000	The vein appeared troubled by air carried along with the water and at a short distance below the cylinder, the space in the centre of the ring disappeared, the cross section changing invariably from an annular to a circular one.
			0·000	
			— 0 050	
33·2105	47·6490	0 6970	— 0·050	
			— 0·050	Vein appeared still troubled by the presence of air within it.
32·8600	47·9557	0·6852	— 0·100	
32·7723	47·8268	0·6852	— 0·100	The vein continues troubled by air.
32·8040	47·7460	0·6855	— 0·100	
32·5971	47·4330	0·6871	— 0·100	
32·4213	47·2098	0·6868	— 0·100	
32·5094	47·8[illegible]	0 6795	— 0·200	The vein always a little troubled by air, but not so much as in preceding experiments.
32.5094	47·8439	0·6795	— 0·200	
32·4215	47·5842	0·6814	— 0·200	
34·8753	47·8269	0·7291	+ 0·005	Air mixed with water, apparently.
34·8763	47·8369	0·7292	+ 0·005	The base of the cylinder 0·005 inch above the plane of the orifice.
34·8753	47·[illegible]	0·7192	+ 0·005	
....	....	....	+ 0·020	
....	....	....	+ 0·020	
....	....	....	+ 0·020	
....	....	....	+ 0·050	
....	....	....	+ 0·050	
....	....	....	+ 0·050	
....	....	....	+ 0·100	The vein yet slightly troubled by air.
....	....	....	+ 0·100	
....	....	....	+ 0·100	
....	....	....	+ 0·200	Vein appears perfectly clear and transparent; no air present in it.
....	....	....	+ 0·200	
....	....	....	+ 0·200	The plane where the presence of the cylinder ceases to effect the discharge, is apparently from 0·25 inch to 0·30 inch above the plane of the orifice.
....	....	....	+ 0·300	
32·0356	47·6328	0·6726	+ 0·300	
....	....	....	+ 0·300	
32·0356	47·6328	0 6726	+ ∞	The cylinder was altogether removed, the vein being perfectly transparent.

EXPERIMENTS ON THE FLOW OF LIQUID, THROUGH ANNULAR SPACES FORMED BY INTRO



Section.

Fig. 4

Plan.

II.—The discharge took place freely in air, under a uniform an orifice in a thin plate, 0·4 inch diameter and the surface orifice tangent to its circumference, to points various distances

$$\frac{\text{Area A B C H G B}}{\text{Area A B C}} =$$

TABLE

1	2	3	4	5	6	7	8		9	10
Series, and Nos. of experiments.	Elevation of water surface in reservoir of supply, A above 0 of hook-gauge scale.	Mean elevation of water surface in reservoir of supply.	Elevation of the plane of the orifice, in a thin plate, A B C referred to 0 of the hook gauge scale.	h=Mean head of wa[illegible]e horizontal orifice A BC.	T Duration of experiments.	Designation of vessels.	Total weight of the vessels with the water contained therein, at the end of each experiment.		D Total mean net discharge.	d Discharge per second in cubic inches $= \frac{1.7315D}{T}$
	inches.	inches.	inches.	inches.	seconds		lbs.	ozs.	ounces.	cub ins.
a 1	3·960		1·016		100	V_1	15	6½	...	...
a 2	3·958	3·958	"	2·942	"	V_1	15	6¼	190·75	3·3028
a 3	3·956		"		"	V_1	15	6¼	...	...
b 4	3·960		"		"	V_1	14	15	...	...
b 5	3·980	3·970	"	2·954	"	V_1	15	0	184·0	3·1859
b 6	3·960		"		"	V_1	14	15¾	...	...
c 7	3·964		"		"	V_1	14	13½	...	...
c 8	3·966	3·964	"	2·948	"	V_1	14	13¾	182·0	3·1513
c 9	3·964		"		"	V_1	14	12½	...	...
d 10	3·998	3·988	"	2·982	"	V_1	14	13	181·5	3·1427
d 11	4·010	4·010	"	2·994	"	V_1	14	13	181·5	3·1427
d 12	3·970	3·970	"	2·954	"	V_1	14	12½	181·0	3·1341
f 13	3·980		"		"	V_1	16	3	...	...
f 14	3·980	3·980	"	2·964	"	V_1	16	3	203·5	...
f 15	3·992		"		"	V_1	16	3	...	...
g 16	3·966		"		"	V_1	17	0	...	...
g 17	3·964	3·964	"	2·948	"	V_1	16	15½	200·25	...
g 18	3·964		"		"	V_1	16	15¾	...	...
h 19	3·960		"		"	V_1	17	10	...	...
h 20	3·960	3·960	"	2·944	"	V_1	17	9	226·66	...
h 21	3 960		"		"	V_1	17	10	...	...
i 22	3·950		"		"	V_1	17	14½	...	...
i 23	3·948	3·948	"	2·932	"	V_1	17	14½	231·0	...
i 24	3·946		"		"	V_1	17	14½	...	...
j 25	3·954		"		"	V_1	18	0	...	...
j 26	3·960	3·956	"	2·940	"	V_1	18	0	232·5	4·0275
j 27	3·956		"		"	V_1	18	0	...	...
28	3 956	3 956	"	2·940	"	V_1	18	0	232·5	4·0275

DUCING A CYLINDRICAL ROD OR DISK INTO A CIRCULAR ORIFICE PIERCED IN A THIN PLATE.

head, through a horizontal lunular space left between the circumference A B C, of G H B, of a cylindrical rod M N O P, 0·185 inch diameter, let down through this K L, above and below its plane Q R. Fig. 4.

$$\frac{0{\cdot}098800}{0{\cdot}125664}=0{\cdot}78622$$

VIII.

11	12	13	14	
Velocity per second $v=\frac{d}{a}$	$\sqrt{2gh}=27{\cdot}78\sqrt{h}$.	Coefficient of velocity or discharge $C_{disch}=\frac{v}{\sqrt{2gh}}$.	Distance K L, between the base M N, of the cylindrical rod and the plane of the orifice + above it, — below it.	**Remarks.**
inches.	inches.		inches.	
....	....	....	0·000	The vein is twisted and troubled by air mixed with water.
33·4295	47·6490	0·7016	0·000	
....	....	....	0·000	
....	....	....	—0·050	
33·2466	47·7137	0·6758	—0·050	
....	....	....	—0·050	
....	....	....	—0·100	Veins twisted and still apparently slightly troubled by air
31·8961	47·6975	0·6687	—0·100	
....	....	....	—0·100	
31·8084	47·9718	0·6631	—0·200	Vein twisted but almost perfectly transparent.
31·8084	48·0682	0·6617	—0·200	
31·7208	47·7460	0·6643	—0·200	
....	....	....	+0·020	Vein twisted and troubled by air.
....	....	....	+0·020	
....	....	....	+0·020	
....	....	....	+0·050	
....	....	....	+0·050	
....	....	....	+0·050	
....	....	....	+0·100	
....	....	....	+0·100	
....	....	....	+0·100	
....	....	....	+0·200	Vein appears to be perfectly transparent.
....	....	....	+0·200	
....	....	....	+0·200	
....	....	....	+0·300	
32·0356	47·6328	0·6726	+0·300	
....	....	....	+0·300	
32·0356	47·6328	0·6726	+∞	The cylinder removed altogether.

EXPERIMENTS ON THE FLOW OF LIQUID THROUGH ANNULAR SPACES, FORMED BY INTRO

Section.
Fig. 5
Plan.

III. The discharge took place freely, in air, under a A B C, of an orifice in a thin plate, 0·482 inch in diameter, thick, fastened to the point of a conical needle, as shown Fig. this orifice to points at various distances K L, above or below

Area of the annular opening A B C I G H = 0·083487

$$\frac{\text{Area A B C I G H}}{\text{Area A B C}} = \frac{0{\cdot}083487}{0{\cdot}182467} = 0{\cdot}4575$$

Ratio of breadth A G, of ring-shaped opening to its

TABLE

1	2	3	4	5	6	7	8		9	10
Series and Nos. of experiments.	Elevation of water surface in reservoir of supply A, above 0 of hook-gauge scale.	Mean elevation of water surface in reservoir of supply.	Elevation of the plane of the orifice, in the thin plate A B C, referred to 0 of hook gauge scale.	h Mean head of water on the horizontal orifice A B C.	T Duration of experiments.	Designation of vessels.	Total weight of the vessels, with the water contained therein, at the end of each experiment.		D Total mean net discharge.	Discharge per second in cubic inches $= d = \frac{1{\cdot}7315D}{T}$
	inches.	inches.	inches.	inches.	seconds.		lbs.	ozs.	ounces.	cubic in.
a { 1	4·036	4·036	1·016	3·020	100	V_I	15	4	188·5	3·2639
2	"		"		"	V_I	15	4		
b { 3	"	"	"	"	"	V_I	15	1	185·5	3·2119
4	"		"		"	V_I	15	1		
c { 5	4·032	"	"	"	"	V_I	14	13½	182·5	3·1600
6	4·040		"		"	V_I	14	14½		
d { 7	4·036	"	"	"	"	V_I	14	15	183·5	3·1773
8	4·036		"		"	V_I	14	15		
e { 9	4·036	"	"	"	"	V_I	15	½	185·0	5·2033
10	4·038		"		"	V_I	15	½		
f { 11	4·038	"	"	"	"	V_I	15	3½	187·5	3·2466
12	4 034		"		"	V_I	15	2½		
13	4·036	"	"	"	"	V_I	17	11½	228·0	3·9478
14	"	"	"	"	"	V_o	24	0	294·0	5 0906
15	"	"	"	"	"	V_o	24	8½	302·5	5·2378
{ 16	"	"	"	"	"	V_o	27	2	344·0	5·9563
17	"		"		"	V_o	27	2		
18	"	"	"	"	"	V_I	15	11½	196·0	....
19	"	"	"	"	"	V_I	16	3½	204·0	....
20	"	"	"	"	"	V_I	16	13	212·5	....
21	"	"	"	"	"	V_I	17	11	227·5	....
22	"	"	"	"	"	V_o	22	13	275·0	....
23	"	"	"	"	"	V_o	24	12	306·0	....
24	"	"	"	"	"	V_o	26	9	338·0	....
25	"	"	"	"	"	V_o	27	2	344·0	....
26	"	"	"	"	"	V_o	27	3	345·0	....

DUCING A CYLINDRICAL ROD OR DISK INTO A CIRCULAR ORIFICE, PIERCED IN A THIN PLATE.

uniform head, through the horizontal annular space left between the circumference and the surface G H I, of a cylindrical disk, 0·355 inch in diameter, and 0·048 inch 5, and let down in the water along the vertical axis J K L, through the centre of its plane Q R.

square inch. Area of orifice A B C = 0·182467 square inch. Hence

mean length D E F, measured in the centre $= \frac{1·3147}{0·0635} = 20·70$

TABLE IX.

10	11	12	13	14	15	
Discharge per second in cubic inches $= \frac{d}{1·7315D}{T}$	Velocity per second $= \frac{v}{d}{a}$	$\sqrt{2gh} = 27·78\sqrt{h}$	Coefficient of velocity or discharge: $C_{disk} = \frac{v}{\sqrt{2gh}}$	Distance K T, between the upper base R S, of the disk and the plane Q R, of the orifice (+ above and — below it).	Distance K L, between the lower base M N, of the disk and the plane Q R, of orifice (+ above and — below it).	Remarks.
cubic in.	inches.	inches.		inches.	inches.	
3·2639	39·0944	48·2765	0·8098	+0·048	0·000	Air apparently mixed with flowing water.
				+0·048	0·000	The lower base M N, in plane Q R, of the orifice.
3·2119	38·4722	"	0·7969	+0·036	—0·012	
				+0·036	—0·012	
3·1600	37·8501	"	0·7840	+0·024	—0·024	
				+0·024	—0·024	
3·1773	38·0575	"	0·7883	+0·012	—0·036	
				+0·012	—0·036	
5·2033	38·3686	"	0·7948	0·000	—0·048	The upper base R S, of the disk, is in the plane Q R, of the orifice.
				0·000	—0·048	
3·2466	17·9786	"	0·37241	—0·012	....	
				—0·012	....	
3·9478	22·7356	"	0·47095	—0·120	....	
5·0906	31·3539	"	0·6495	—0·240	....	The axis of the vein continues to coincide with the vertical through the centre of the orifice.
5·2378	33·2558	"	0·6888	—0·340	....	Vein quite transparent; no air present; area of annular space in plane Q R $= a =$ 7854-$(·482^2—·178^2) = ·2005 \times 7854 = 0·1575$ square inche.
5·9563	32·6435	"	0·6762	....	....	The disk removed altogether.
....	....	....	....	....	+0·012	
....	....	....	....	....	+0·024	
....	....	....	....	....	+0·036	
....	....	....	....	....	+0·048	
....	....	....	....	....	+0·096	
....	....	....	....	....	+0·144	
....	....	....	....	....	+0·244	
....	....	....	....	....	+0·312	At this elevation the presence of the disk ceases apparently to influence the discharge sensibly
....	....	....	....	....	....	The disk removed altogether.

EXPERIMENTS ON THE FLOW OF LIQUID, THROUGH ANNULAR SPACES FORMED BY INTRO

IV. The discharge took place freely, in air, under a uniform head, through the 0·384 inch in diameter, and the surface of a cylindrical disk, 0 355 inch in diameter, ceding page, in case III, and let down along the vertical passing through the centre

Area a, of the annular passage $= 0·016832$ square inch—Area o, of the complete

$$\frac{a}{o} = \frac{0·016832}{0·115812} = 0·1453.$$

Ratio of breadth of ring to its mean length measured in the centre $= \dfrac{1·1650}{0·0145}$

TABLE

1	2	3	4	5	6	7		8	9
Nos. of experiments.	Elevation of water surface in reservoirs of supply A, above 0 of hook guage scale.	Elevation of the plane of the orifice in the thin plate A B C, referred to 0 of hook guage scale.	h Mean head of water on the horizontal orifice, A B C.	T Duration of experiments.	Designation of vessels.	Total weight of the vessels with the water contained therein, at the end of each experiment.		D Total mean net discharge.	d = discharge, per second, in cubic inches $= \dfrac{1·7315\,D}{T}$
	inches.	inches.	inches.	seconds.		lbs.	oz.	ounces.	cubic in.
1	3·942	0·832	3·110	300	VII	10	14	127·25	0·73444
2	3·942	"	"	"	VII	10	14	127·25	0·73444
3	3 942	"	"	"	VII	9	11	108·25	0·62478
4	3·942	"	"	"	VII	9	10	107·25	0·61901
5	3·922	"	3·090	"	VII	9	9	106·25	0 61324
6	3·942	"	3·110	"	VII	9	11	108 25	0 62478
7	3·942	"	"	"	VII	10	8	121 25	0·69981
8	3·942	"	"	"	VII	10	12½	125 75	0·72579
9	3·932	"	3·100	"	VII	11	½	129·75	0·74887
10	3·932	"	3·100	"	VII	11	½	129·75	0·74887
11	3·942	"	3·110	"	VII	11	2	131·25	0·75753
12	3·942	"	"	100	VI	12	14	150·50	2·60591
13	3·942	"	"	100	VI	14	10½	179·00	3·0994
14	3·942	"	"	300	VII	10	15	128·25	....
15	3·942	"	"	300	VI	19	15	263·50	....
16	3·952	"	3·120	100	VI	14	3	171·50	....
17	3·952	"	"	"	VI	16	6	206 50	....
18	3·952	"	"	"	VI	17	0	216·50	3·7487
19	3·952	"	"	"	VI	17	0	216·50	3·7487
20	3·952	"	"	"	VI	17	0	216·50	3·7487

horizontal annular space left between the circumference of an orifice in a thin plate, and 0·048 inch thick, stuck on the point of a conical needle, as shown on the pre- of this orifice to points at various distances above and below its plane.
circular orifice = 0·115812 square inch—whence the ratio between the two areas =

= 80·35.

X.

10	11	12	13	14	
v, velocity, per second $= \frac{d}{a}$	$\sqrt{2gh} = 27\cdot78\sqrt{h}$	Coefficient of velocity or discharge, $C_{disch} = \frac{v}{\sqrt{2gh}}$	Distance K T, between the upper base R S, of the disk, and the plane Q R of the orifice (+ above, and — below it.)	Distance, K L, between the lower base M N, of the disk, and the plane Q R of the orifice (+ above, —below it.)	Remarks.
inches.	inches.		inches.	inches.	
43·6338	48·9906	0 8907	+ 0·048	0 000	The under side of the disk is in the plane of the orifice. Vein troubled by air mixed with flowing water.
43·6338	"	0·8907	+ 0 048	0·000	
37·1187	"	0·7577	+ 0·036	— 0·012	
36·7759	"	0 7507	+ 0 032	— 0 016	Vein apparently still somewhat troubled by air—in experiments Nos. 3, 4, 5, 6, but not so much as in experiments Nos. 1 and 2.
36 4330	48 8327	0·7461	+ 0·032	— 0·016	
37·1187	48 9906	0·7577	+ 0·028	— 0 020	
41 5764	"	0·8487	+ 0 008	— 0·040	In all the experiments from No. 1 to No. 12, the liquid fillets meet, in the axis passing through the centre of the orifice approximately at a distance of from ¼ to ⅓ inch below the orifice.
43 1195	"	0·8802	+ 0 002	— 0·046	
44·4911	48·9117	0·9096	0·	— 0 048	
44·4911	"	0 9096	0·	— 0 048	Vein rendered somewhat opaque by air carried along by water, in experiments Nos. 8, 9, 10, 11, about to the same extent as in experiments Nos. 1 and 2.
45·0054	48·9906	0 9187	0·	— 0·048	
24·0353	"	0·4906	— 0·096	....	
31 2123	"	0·6371	— 0 184	....	Vein much clearer, apparently, than in any experiment between Nos. 1 and 12.
....	....	....	....	+ 0 008	
....	....	....	....	+ 0·028	
....	....	....	....	+ 0·096	
....	....	....	....	+ 0·192	
32·3688	49 0693	0 6596	....	+ 0·244	Vein perfectly clear.
32 3688	"	"	....	+ 3 198	Vein perfectly transparent.
32.3688	"	"	....	....	The disk removed altogether.

EXPERIMENTTS ON THE STEMMING POWER OF THE NATURALLY CONTRACTED VERTI

from the reservoir of supply S into a receiving vessel R, under a pressure of
Fig. 6½, between 19 and 20 minimum diameters of 0·305 inch, or 5–8 to 6 inches long,

TABLE

1	2	3	4	5	6	7	8	9	
	No. 1—Vertically descending vein V, issuing from horizontal orifice O, 0·482 inch diameter in thin plate, for which coefficient of velocity of efflux = 0 680 under a pressure of 3 inches, or 50.						No. 2—Vein V, projected through orifice O, 0·420 inch diam., for which coefficient of velocity of efflux = 0·677.		
	H_1	H_2	$\frac{H_2}{H_1}$	d	a	$\frac{a}{0 \cdot 07360}$	H_1	$\frac{H_2}{H_1}$	
Letter for reference.	Total fall from water surface of reservoir of supply S to orifice and minimum cross section E of divergent receiving tube T.	Total rise of water in receiving reservoir R above the plane EF of the inlet orifice, and minimum cross section E of divergent tube T, before any lateral spilling commenced.		Diameter of vein V at plane EF of inlet orifice of divergent tube T, according to direct measurements made with points mounted on a diaphragm.	Area of cross section of naturally contracted descending vein V at plane EF of inlet orifice of the divergent tube T.	Ratio of area a of naturally contracted descending vein V to area 0·07360 sq. inch of orifice or minimum cross section of divergent tube T.			
	Inches.	Inches.		Inches.	Sq. inches.		Inches.		
A	3·65	......	......	......	......	......	......	......	......
B	4·15	......	......	......	......	......	......	......	......
C	5·15	......	......	......	......	......	......	......	......
D	6·15	......	......	......	......	......	......	......	......
E	7·15	......	......	......	......	......	......	......	......
F	7 65	......	......	......	......	......	......	......	......
G	8·15	......	......	......	......	......	......	......	......
H	8·65	4·15	0·509	0·314	0·07744	1·0600	5·15	0·632	......
I	9·15	......	......	......	......	......	6·15	0·672	......
J	9·65	......	......	......	......	......	6·65	0·689	......
K	10·15	7·00	0·689	0·293	0·06743	0·9229	7·05	0·695	......
L	10 65	......	......	......	......	......	7 30	0·686	......
M	11·15	......	......	......	......	......	......	......	......
N	12·15	8·55	0·703	0·286	0·06424	0·8728	8·20	0·675	......
O	13·15	9·15	0·696	0·280	0·06158	0·8366	......	......	......
P	14·15	9·95	0·703	0·275	0·05940	0·8070	......	......	......
Q	15·15	10·75	0·709	0·269	0·05683	0·7721	......	......	......
R	16·15	11·45	0·709	0·264	0·05474	0·7437	......	......	......
S	17·15	12·15	0·703	0·259	0·05269	0·7159	......	......	......
T	20·15	14·15	0·702	......	......	......	13·00	0·645	......

CALLY DESCENDING VEIN V, PASSING THROUGH A SIMPLE ORIFICE O IN A THIN PLATE, 3 inches = M N, through a trumpet mouth shaped divergent tube, shown full size, and provided with a short conoïdal convergent entrance E. (See Fig. 6)

XI.

10	11	12	13	14	15	16	17	
No. 3—Vein V, projected through orifice 0.400 in. diam., for which coefficient of velocity of efflux = 0.670.					No. 4—Vein V, projected through orifice 0·348 in. diam., for which coeff. velocity of efflux = 0·654.		Letter of reference.	
H_2	$\frac{H_2}{H_1}$	d	a	$\frac{a}{0·07360}$	H_2	$\frac{H_2}{H_1}$		
Inches		Inches	Sq. in.		Inches.			Remarks.
1·35	0·370	0·317	0·07892	1·0802	1·80	0·493	A	
1·80	0·433	0·301	0·07116	0·9739	2·50	0·602	B	
3·25	0·631	0·287	0·06469	0·8854	3·15	0·611	C	
....	....	....	....	....	3 90	0·634	D	
4·80	0·671	0·262	0·05391	C·7379	4·60	0·643	E	
5·30	0·693	0·258	0·05228	0·7155	5·00	0·654	F	
5·65	0·693	0·252	0·04987	0·6826	5·45	0·669	G	Ratio $\frac{H_2}{H_1}$ max for veins Nos. 3 and 4
5·90	0·682	0·248	0·04830	0·6612	....	....	H	
6·15	0·672	0·244	0·04676	0·6400	6·10	0·667	I	
....	....	....	....	....	....	....	J	
6·85	0·675	0·239	0·04486	0·6140	6·60	0·650	K	Ratio $\frac{H_2}{H_1}$ a maximum for vein No. 2
....	....	....	....	....	....	....	L	
7·50	0·672	0·234	0·04300	0·5886	7·30	0·654	M	
....	....	....	....	....	....	....	N	
....	....	....	....	....	....	....	O	
9·25	0·653	0·222	0·03871	0·5298	8·65	0·611	P	
....	....	....	....	....	....	....	Q	Ratio $\frac{H_2}{H_1}$ a maximum for vein No. 1
....	....	....	....	....	....	....	R	
10·85	0·632	0·210	0·03464	0·4741	10·00	0·583	S	
12·15	0·603	0·204	0·03268	0·4474	11·20	0·556	T	

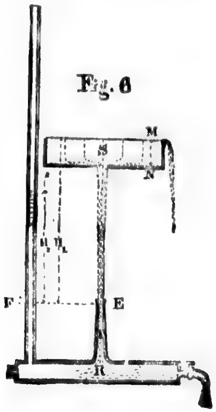


Fig. 6

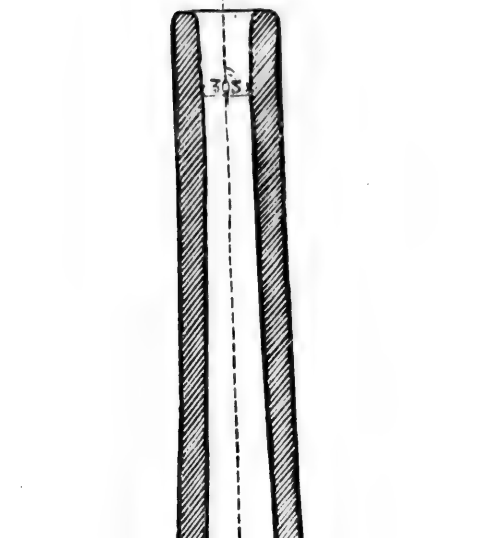


Fig. 6½

EXPERIMENTS on the efflux of water in the open atmosphere through circular orifices, A B, in thin walls, whose sides, E A, F B, form with the axis I X, passing through the centre of the orifice perpendicularly to its plane, angles, E O I, F O I, greater than a right angle, on the inside of the reservoir. (See fig. 7.)

EXPERIMENTS on the efflux of water in the open atmosphere through circular orifices, A B, in thin walls, whose sides, E A, F B, form with the axis I X, passing through the centre of the orifice perpendicularly to its plane, angles, E O I, F O I, greater than a right angle, on the inside of the reservoir. (*See fig.* 7.)

TABLE XI½.

1	2	3	4	5	6	7	8	9		10	11	12	13	14
No. of Experiment.	Diameter A B of orifice.	Angle E O I or F O I.	Elevation of water surface, G H, in reservoir of supply, A, above 0 of hook gauge scale.	Elevation of the plane of the orifice AB, above 0 of hook gauge scale.	h.—Mean head of water on the horizontal orifice A B.	T. Duration of experiments.	Designation of vessels.	Total weight of the vessels with the water contained in them at the end of each experiment.		D Total mean net discharge.	d Discharge per second, in cubic inches $= \frac{1.7318\,D}{T}$	v, Velocity per second $= \frac{d}{a}$	$\sqrt{2\,gh} = 27.78\,\sqrt{h}$	Coefficient of velocity or discharge: $C_{disch} = \frac{v}{\sqrt{2\,gh}}$
	inches	deg's.	inches.	inches.	inches.	sec's		lbs.	ozs.	ozs.	cub. in.	inches.	inches.	
1	0·405	135°	3·180	0·560	2·62	100	V1	17	½	217	3·7573	29·5309	44·9480	0·6570
2	0·405	135°	3·210	0·560	2·65	100	V1	17	4½	221	3·8266	30·0752	45·2225	0·6650
3	0·416	157½°	4·620	2·090	2·530	200	Vo	31	7½	413½	3·5759	26·2934	44·1869	0.5950
4	0·416	157½°	4·564	2·090	2·474	200	Vo	31	4½	410½	3·5539	26·1316	43·6970	0·5980
5	0·416	157½°	4·568	2·090	2·478	200	Vo	31	5	411	3·5582	26·1632	43·7340	0·5980

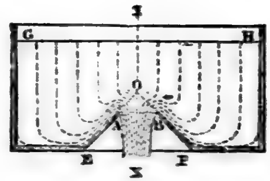


Fig. 7

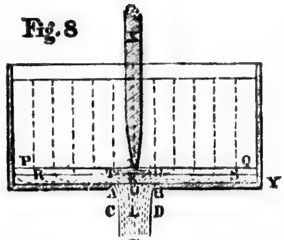
REMARKS.

Area of orifice A B (*fig.* 7), = 0·127235 square inch. This orifice was not strictly one with a sharp edge; the thickness of the metallic rim around it, measured in the plane A B, was = 0·015 inch; it is no doubt for this reason that the coefficient of efflux, or velocity in the orifice, turned out as high as 0·6570.

This orifice, whose area was 0·136 square inch, had a very sharp edge; its plane, A B, stood 0·47 inch above the bottom E F of the reservoir, whose interior diameter, as already stated, was 12 inches; temperature of the air = 70° Fahrenheit, water 52°.

THEORY.

Fig. 8



Let H represent the head of water, —— , on the orifice A B, in a thin plate;

r, the radius, A O = O B of the orifice A B;

y, the radius, C E = E D, of the cross-section C E D, taken at any point, E;

x, the distance E O of the point E, from the centre O, of the orifice;

$d\,x$, an increment of length of the vein;

$V_{orif.}$, the velocity of the liquid, in the plane of the orifice, A B;

v, the velocity of the water, at any point, E, on the axis of the vein;

g, the acceleration of gravity, per second;

γ, the heaviness of water, or weight of one unit of volume.

We know, from experiments with liquid jets, from, say $\frac{1}{2}$ inch in diameter upwards, produced under various heads, up to, say 10 feet, as a matter of fact, that if a jet or vein of water is interrupted at any point whatsoever, the last particles of liquid immediately in front of the interrupting body rise as high, vertically, and reach as far, horizontally, in vacuo or even in the open air, as if the continuity of the vein had not been broken.

We may therefore take for granted, that the whole energy e, which the hydrostatic pressure exerted on the top covering or sides of a reservoir, is capable of developing, through a given orifice A B, in the unit of time, is invariably imparted to the spurting water within the reservoir, before the liquid particles pass the plane of that orifice, and the assumption that this is also the case for vertically descending veins, projected through orifices in the horizontal bottom of a reservoir is not unreasonable. Hence, if gravity be abstracted outside of the reservoir of supply, the measure of an element, de of this energy, must be the same for all sections of one and the same vein.

But, in general, the amount of energy e, stored in any moving mass is represented by the product of the square of the velocity v, the volume of the body, and its heaviness γ, divided by twice the acceleration of gravity, viz., $2g$; we must, therefore have, in any theoretically perfect liquid circular jet, uninfluenced by gravity after leaving the orifice, the relation.

$$de = \frac{v^2}{2g}\pi r^2 dx\gamma = \left\{ \begin{array}{l} \text{Constant quantity for every elementary slice or} \\ \text{sheet of liquid contained in the vein.} \end{array} \right.$$

Whence it follows, that in general:

$$de = \frac{v^2}{2g}\pi r^2 dx\gamma = \frac{V^2_{orif}}{2g}\pi r^2 dx\gamma$$

by considering $\pi r^2 dx$ to be the increment of the volume of liquid discharged or ejected from the reservoir of supply, during the unit of the time t, which corresponds to dt.

Now I found, by direct measurement (See Table III):

1. That the area of the section of greatest contraction of a liquid circular vein projected vertically downward, through an orifice 0·4 inch in diameter, under a constant head of about 3 inches is:

$$\pi r^2_{\text{cont.}} = 0{\cdot}6610\,\pi r^2\,;$$

$r_{\text{cont.}}$ standing for the radius of the circular perimeter at the section of maximum contraction.

2. That the square of the velocity, ($V_{orif.}$) in the plane of the orifice is:

$$V^2_{orif.} = (0{\cdot}6662)^2\,2gH = (0{\cdot}4438)\,2gH.$$

Whence, admitting that in a perfectly liquid stream, or in a continuous stream of infinitely small sensibly equidistant bodies, the velocity must vary inversely as the

area of the vein; we obtain at the section of maximum contraction, for the square of the velocity, $v_{\text{cont.}}$

$$v^2_{\text{cont}} = 2g\text{H}\ (\cdot 4438) \left(\frac{\pi r^2}{\cdot 6610 \pi r^2}\right)^2 = 2g\text{H} \left(\frac{0\cdot 4438}{0\cdot 4376}\right) = 1\cdot 0157\ (2g\text{H}).$$

We must therefore necessarily have, for the energy of every element of volume of the liquid vein, under consideration:

$$de = 1\cdot 0157\ \text{H}\,\pi r^2 dx\gamma = \cdot 4438\ \text{H}\,\pi r^2 dx\gamma.$$

Now this result is clearly impossible or absurd of itself, and cannot obtain unless we admit:

That in the plane of the orifice A B, the intensity $i_{\text{orif.}}$ of the moving force is less than that $i_{\text{cont.}}$ at the section of maximum contraction, in the ratio of 0·4438 to 1 0157 and increases gradually from the former to the latter place, whether or not the vein be interrupted at any point, whence we are led to the conclusion that: $i_{\text{orif.}} = \cdot 4369\ i_{\text{cont.}}$, or $i_{\text{cont.}} = 2\cdot 2885\ i_{\text{orif.}}$ must obtain either on account of the mutual interference of the jammed up liquid particles, or in consequence of some other corresponding molecular action or owing to a combination of some such actions.

Again, Table IV shows that for a vertically descending vein projected through a circular orifice 0·482 inch in diameter, under a head of 3 inches:

$$\pi r^2_{\text{cont.}} = 0\cdot 6972\ \pi r^2 \qquad \text{and}$$

$$\text{V}^2_{\text{orif.}} = 2g\text{H}\ (\cdot 6803)^2 = \cdot 4628\ (2g\text{H}) \qquad \text{hence:}$$

$$v^2_{\text{cont.}} = \cdot 4628\ (2g\text{H}) \left(\frac{\pi r^2}{\cdot 6972 \pi r^2}\right)^2 = 2g\text{H} \left(\frac{\cdot 4628}{\cdot 4861}\right) = 0\cdot 9521\ (2g\text{H})$$

whence:

$$de = 0\cdot 9521\ \text{H}\,\pi r^2 dx\gamma = 0\cdot 4628\ \text{H}\ (\pi r^2)\ dx\gamma, \qquad \text{and}$$

$$i_{\text{cont.}} = \left(\frac{0\cdot 9521}{0\cdot 4628}\right) i_{\text{orif.}} = 2\cdot 0573\ i_{\text{orif.}} \qquad \text{or,}$$

$$-i_{\text{orif.}} = 0\cdot 4860\ i_{\text{cont.}}$$

Finally, by adding, in the table of experimental results, recorded by Michelotti, the younger, which is given in Spon's Dictionary of Engineering, p. 1891, a column of coefficients of velocity in the orifice $C_{\left(\frac{\text{vel.}}{\text{orif.}}\right)}$ and one of ratios, $\left(\frac{i_{\text{orif.}}}{i_{\text{cont.}}}\right)$, based on the measurements made by the author just named, we obtain:

TABLE XII.

Letter of reference.	Head on the orifice in english feet.	Diameter of the vein in inches.		Ratio between the diameter or radius at the orifice, and that at the section of maximum contraction. $\frac{r_{\text{cont.}}}{r}$	Coefficient of velocity in the orifice. $C_{\left(\frac{\text{vel.}}{\text{orif.}}\right)} = \frac{V_{\text{orif.}}}{\sqrt{2gh}}$	Approximate ratio between the respective intensities, i_{orif} and $i_{\text{cont.}}$ of the moving force in the plane of the orifice and at section of maximum contraction. $\frac{i_{\text{orif.}}}{i_{\text{cont.}}} = \frac{r^4_{\text{cont.}}}{r^4}$
		At the orifice.	At the section of greatest contraction.			
A	6·890	6 394	5 047	0·790	0·691	0·3091
B	12·008	6 394	5·039	0 788	0 691	0 3861
C	7,349	3·197	2·511	0·786	0·613	0·3817
D	12·502	3·197	2·504	0·783	0 612	0·3751
E	22·179	3·197	2·413	0·755	0 597	0 3247

It is plain, judging by the results arrived at, that the ratio $\frac{i_{\text{orif.}}}{i_{\text{cont.}}}$ is not constant for all veins, but that it increases simultaneously with the area of the orifice and

diminishes as the head increases, or else, that the complete vein, R A C D B S, protrudes more through the orifice A O B, in some cases than in others. Possibly the variations of this ratio, as exhibited in Table XII., are governed conjointly by the intensity of the pressure, the area of the orifice and the position of the entire vein, in reference to the plane of this orifice.

There is nothing to show however, as yet, why in one and the same perfectly fluid vein, the variations in the intensity of the force, by virtue of which the liquid acquires motion and the final energy is generated, should be different during the time of describing the last increment of the portion of the trajectory outside of the reservoir between the orifice and the section of maximum contraction, viz: that nearest to the section just named, wherever that may be situated, from what it is, while an increment of trajectory is described by the liquid, close to the orifice A O B, or even at any point of the vein within the reservoir back of this orifice. Neither is there any thing to indicate why one or the other of the respective intensities, $i_{cont.}$, $i_{orif.}$ should prevail at one time rather than at any other time, during the progress or formation of one and the same vein.

Hence, there is good reason for concluding that $i_{orif.}$ and $i_{cont.}$ truly represent the alternating intensities of two forces, $fi_{orif.}$ and $fi_{cont.}$ which govern the motion of every contracted fluid vein, both within and without the reservoir, and that $\frac{i_{orif.}}{i_{cont.}}$ is the ratio of two sensibly uniform accelerations generated alternately, each during an increment of time, dt in every one of the elementary fluid sheets of which any vein may be considered to consist.

From a theoretical point of view, all extraneous resistances and forces being abstracted, gravity included, any unopposed liquid vein, once generated, must evidently continue its course over an infinite distance beyond the orifice, outside of the reservoir, whence it draws its supply; and the time consumed in describing this portion of its path must be infinitely great in all cases. On the inside, however, of this reservoir, the vein can extend only up to the point where on account of the moving force acting with the alternate intensities, $i_{cont.}$, $i_{orf.}$ upon a very great or say infinite number of liquid molecules embraced in its field of action—motion becomes impossible or infinitely small, comparatively speaking. The position of the plane where the vein ceases to exist as such, or rather properly commences within the reservoir, viz., the position of the plane of rest, may be considered to be affected by the volume of liquid discharged through a given orifice, in the unit of time, only in so far as the hydraulic pressure modifies the conditions of the molecular structure of the liquid.

Again, although it is quite true that in every complete and permanently established vein the liquid is continually passing from a less to a greater velocity, both in and outside of the reservoir, nevertheless the velocity $v_{orif.}$ proper to the plane of the orifice, cannot, for one reason or for another, be attributed to the action of the force $fi_{orif.}$ in preference to that of the force $fi_{cont.}$

Keeping therefore in view, that in every perfectly fluid vein the areas of the cross-sections must, of necessity, vary inversely as the total velocities generated from a state of rest in the corresponding elementary sheets of liquid moved forward simultaneously, the volume of each of which may be represented by $\pi r^2 dx$, it becomes apparent that in order that the stream may embrace a circular section of the requisite area, to fill or cover the entire orifice in the thin plate, equally well when we consider the total acceleration, viz.: that which corresponds to the actual permanent velocity acquired by the fluid—to be generated by the moving force while its intensity is $i_{cont.}$ as when the same total acceleration or velocity is considered to be generated by the said moving force while its intensity is but $i_{orif.}$ (taking now for granted that $fi_{o.}$ and $fi_{c.}$ are [redacted] constant,) an indispensable condition is, that the time during which the force $fi_{c.}$ acts should bear to the time during which the force fi_{o} is acting on each elementary volume of liquid ejected—while the stream passes from a state of rest within the reservoir to the orifice A O B, the ratio of $i_{orif.}$ to $i_{cont.}$, that is to say: the ratio of 1 to 2·2, or thereabouts. For it is only in such case that the rates of motion corresponding to the total numbers or sums of increments of acceleration generated from a state of rest by each one of the

forces $fi_{c.}$ and $fi_{o.}$, or the sums of the increments of the gradual retardations due to the lateral extension of the vein under the government of the said forces, are precisely equivalent at the plane of the orifice after the liquid stream has assumed its definitely permanent state.

Let us now devote some attention to the consideration of the molecular structure of fluid matter in connection with the subject under discussion.

In a paper entitled: "On a Fourth State of Matter,"* which was read by Prof. J. W. Crookes, F.R.S., before the Royal Society of Great Britain, on the 10th of June, 1880, this *savant* explained what seemed to him to be the constitution of matter in its three states, of solid, liquid and gas. In the views which he expressed there appears to be embodied all that is at the present time generally known and admitted in this connection.

The structure of all solid and liquid matter appears to be as follows (using Mr. Crook's own words):

"Solids as well as liquids are composed of discontinuous molecules, separated from each other by a space which is relatively large—possibly enormous—in comparison with the diameter of the central nucleus we call *molecule.* The molecules themselves built up of *atoms,* are governed by certain forces. Two of these forces are attraction and motion. Attraction, when exerted at sensible distances, is known as gravitation, but when the distances are molecular it is called *adhesion* and *cohesion.* Attraction appears to be independent of absolute temperature; it increases as the distance between the molecules diminishes; and were there no counteracting force, the result would be a mass of molecules in actual contact, with no molecular movement whatever—a state of things beyond our conception—a state, too, which would probably result in the creation of something that, according to our present views, would not be matter."

"This force of cohesion is counterbalanced by the movements of individual molecules themselves, movements varying directly with the temperature, increasing and diminishing in amplitude as the temperature rises and falls.

"The molecules in solids do not travel from one part to another, but possess adhesion and retain fixity of position about their centres of oscillation. Matter, as we know it, has so high an absolute temperature that the movements of the molecules are large in comparison with their diameter, for mass must be able to bear a reduction of temperature of nearly 300° C. before the amptitude of the molecular excursions would vanish.

"The state of solidity, therefore—the state which we are in the habit of considering *par excellence* as that of *matter*—is merely the effect on our senses of the motion of discrete molecules among themselves.

"Solids exist of all consistencies, from the hardest metal, the most elastic crystal, down to the thinest jelly. A perfect solid would have no viscosity, *i.e.*, when rendered discontinuous or divided by the forcible passage of a harder solid, it would not close up behind and again become continuous.

"In solid bodies the cohesion varies according to some unknown factor, which we call chemical constitution; hence, each kind of solid matter requires raising to a different temperature before the oscillating molecules loose their fixed position with reference to one another, at this point, varying in different bodies, the solid becomes liquid.

"In liquids the force of cohesion is very much reduced, and the adhesion or fixity of position of the centers of oscillation of the molecules is destroyed. When artifically heated the inter-molecular movements increase in proportion as the temperature rises until, at last, cohesion is broken down and the molecules fly off into space with enormous velocities.

"Liquids possess the property of viscosity—that is to say, they offer a certain opposition to the passage of solid bodies: at the same time they cannot permanently resist such opposition, however slight, if continuously applied. Liquids vary in consistency from the hard, brittle and apparently solid pitch, to the lightest and most etherial liquid capable of existing at any particular temperature.

*See page 3796, No. 238, Vol. X., *Scientific American* Supplement, July 24, 1880.

"The state of liquidity is therefore due to inter-molicular motions of a larger and more tumultuous character than those which characterize the solid state."

From the constitution, or molecular structure, of liquids, as just described, it follows that any effort at separating and moving away in any direction, one elementary layer or sheet of molecules from the next succeeding one and the general body of a liquid stored in a reservoir, must necessarily overcome during an infinitesimal space of time, in addtion to the inertia of the fluid matter, also a part of its cohesion—within the limits of the sphere of molecular oscillations of attraction and repulsion—before the total increase of motion or acceleration which ~~[illegible]~~ is capable of imparting to the fluid particles, viewed as independent solid bodies can be fully developed. This condition of liquid motion I take to be corroborative of the reality of the alternating intensities $i_{orif.}$ and $i_{cont.}$ of the moving force, the existence of which was previously deduced directly from the indications afforded by the experimental enquiries.

viz: this effort ust undergo he influence

When, owing to lateral communication among the liquid molecules, proceeding from the orifice A O B in the thin plate towards the interior of the reservoir, the field of action embraced by the pressure on the area of this orifice has become enlarged to such an extent that the rate of separation of liquid sheets from the main body has become infinitely slow, it is clear that the origin K, of the ultimate motion existing at the section of maximum contraction, is reached; but the plane of rest P Q, as regards solicitation of the liquid particles by the force fi_o in the direction O E within the sphere of mutual attraction, must lie yet some distance further back of the plane of this orifice, viz., at a point N, where all disturbance in the oscillations of the molecules of the fluid which correspond to its temperature ceases, or where the said force, fi_{orif} must commence to actin order that the requisite separation of a sheet of liquid from the main body may be completely effected at the plane R S.

We have just seen, judging from indications furnished by the experiments made, that in every liquid vein the permanent motion is apparently the result of two alternating forces fi_o and fi_c acting upon an invariable elementary volume of water corresponding to the area of the orifice or of one constant force applied against the varying resistances offered alternately by the said elementary volume of water, during a space of time such as to allow of the same velocity being generated by each force, during the passage of the liquid from the plane where the forward movement originates within the reservoir to the orifice A O B. Up to this stage the two forces fi_o and fi_c were assumed to be absolutely constant, according to the constitution of liquids, as above quoted, however, attraction or cohesion decreases as the distance between the molecules increases; furthermore, it does not seem improbable that the degree of separation of every two consecutive elementary layers or sheets of molecules of a liquid stream is, in some measure, directly proportional to its velocity—whence it follows: that fi_o and fi_{cont} may vary simultaneously with the velocity of the vein.

Notwithstanding the possible variable character of fi_o and fi_c, there is nothing preventing us—with a view of rendering the artifices of computation less complex, and the mental processes involved easier to follow—to consider fi_o and fi_c as denoting the mean values of these forces between any two limiting planes we may choose, as say for instance, between the planes R S and A O B, within the reservoir, or A O B and C E D, outside of it.

Let us now suppose, that by introducing into the water, back of the orifice A O B, a disk or other solid body T U, the area of whose cross-section is small in comparison to that of the reservoir, we determine approximately, or else that we succeed in establishing theoretically or experimentally by some other means—with greater accuracy, if possible—the distance O N=s, at which the conditions of molecular equilibrium would cease to be affected by the flow of liquid through the orifice A O B, if the area of the cross-section of the reservoir, taken in a direction P Q, parallel to the plane of this orifice, was very great—and where, therefore, the presence of a solid body would not diminish the volume of liquid discharged in a given time under a given head. Then, no matter what may be the absolute length of time during which the moving force may have to act, from the instant of opening the orifice A O B, to the establishment of permanent motion and the definite forma-

tion of the vein—leaving friction and all secondary resistances out of consideration for the moment—this distance $s=$O N, may be considered to be the actual space described during the time just mentioned, by an elementary sheet of liquid solicited exclusively by the mean, lesser or reduced variable force fi_o regarded as being constant—and $\frac{i_o}{i_c}s=$O K, the space described by a sheet of liquid subjected to the greater force fi_c with the mean velocity proper, as regards this force, to the portion of vein lying between the orifice A O B, and a plane R K S, where the separation of liquid particles from the main body within the reservoir and from one another, ceases as we proceed, from the orifice inwards, or commences, going in a contrary direction, and takes place at an infinitely slow rate. That is to say, the distance O K, between the plane A O B, and a plane R K S, whence a body solicited uniformly by the force fi_c regarded as constant—with a mean acceleration—would have to start in order to pass the plane A O B, with a velocity equal to that which the same body would have after passing over the distance N O, under the influence of the force fi_c is equal to $\frac{i_o}{i_c}s$,—for $\sqrt{i_o}$ correctly represents the mean velocity generated in any body by the lesser force fi_o while the greater force fi_c generates in the same body, the equivalent mean velocity corresponding to $\sqrt{\frac{i_o}{i_c}\times i_c\, s}$. Or, if it be thought preferable to assume that the motions due to the two forces fi_o and fi_c, have simultaneous initial or final instants, and begin or cease at the same plane, within the reservoir—then, in order that equal velocities may be generated by both these forces in the elementary volume of liquid ejected, from the plane of rest and origin of motion up to the orifice A O B, it is necessary that the larger force fi_c should act during a shorter space of time than the smaller one fi_o, viz : so as to cause the virtual space $\frac{i_o}{i_c}s$, to be described, while under the influence of the latter, the space s, is gone over In either case the result is the same.

Furthermore, following the same line of argument, it is plain that at any distance O E$=x$ from the centre O, whether measured within or without the reservoir along the axis E O K X of the vein, the final velocity generated by the force $f\,i_c$ during the interval of time which elapses, after the establishment of permanent motion, between the passage of an elementary volume of liquid at the plane R S and the passage of the same elementary slice at any other section C E may properly be represented by $\sqrt{i_o s+i_o x}$ and also that the total amount of acceleration generated by the force $f\,i_c$ while the space K E$=$K O$+$O E is described by the vein in its permanent condition may be represented by the expression $\sqrt[4]{i_c\left(\frac{i_o}{i_c}\right)s+{}_cx}=\sqrt{i_o s+i_c x}.$

Now the increment of volume moved forward successively at every instant remains clearly invariable so long as the pressure in the reservoir is kept at a uniform intensity; the vein having to lengthen out sufficiently at every step to provide room for each new accession to its fold. Therefore, since the sum total of the increments of acceleration generated by the moving force while overcoming both the inertia and unimpaired cohesion of the liquid particles, must also bear to the sum of the increments of acceleration accumulated while this force has to contend merely against the inertia of matter as reduced by repulsion the unceasingly varying mean ratio of $\sqrt{i_o s+i_o x}$ to $\sqrt{i_o s+i_c x}$ in order that both these conditions may be fulfilled simultaneously there remaims no alternative but for the areas of the cross-sections of the vein to vary inversely as this ratio, viz., we must have always :—

$$\pi y^2=\pi r^2\times\frac{\sqrt{i_o s+i_o x}}{\sqrt{i_o s+i_c x}}$$

As nothing definite is known concerning the laws which govern the variations of the ratio of i_o to i_c, in order to simplify this formula and all others based thereon, let us divide both the numerator and denominator of the fraction in the second member of this equation by i_c and also by π and further substitute i for $\frac{i_o}{i_c}$ —when we obtain

$$y^2 = r^2 \frac{\sqrt{is+ix}}{\sqrt{is+x}} \tag{a}$$

whence we deduce, for the fundamental equation of the curve whose revolution about the axis E X generates a conoid similar to the theoretical naturally contracted fluid vein A O B D E C, abstracted from gravity:

$$y = r \frac{\sqrt[4]{is+ix}}{\sqrt[4]{is+x}} \tag{b}$$

Now granting—as many experiments made with jets of medium sizes, produced under heads or pressures, neither very small nor very great, tend to prove—that the energy generated per unit of volume of the liquid issuing from an aperture in a reservoir under ordinary conditions of flow, is in general proportional to these heads, and denoting by $\left(\frac{\text{coeff. vel. head}}{\text{AOB}}\right)$ the ratio $\frac{V^2\left(\frac{\text{orif.}}{\text{AOB}}\right)}{H}$ between the head due to the actual or experimental velocity of efflux $V_{\left(\frac{\text{orif.}}{\text{AOB}}\right)}$ and the head H=O X the total height of liquid pressing on the orifice A O B, we have for the velocity at this orifice:

$$V_{\left(\frac{\text{orif}}{\text{AOB}}\right)} = \sqrt{2g\left(\frac{\text{coeff. vel. head}}{\text{AOB}}\right) H}$$

whence we deduce for the velocity v_{CED} at any section C E D:

$$v_{\text{CED}} = \frac{\sqrt{2g\left(\frac{\text{coeff. vel. head}}{\text{AOB}}\right) H(x+is)}}{\sqrt{is+ix}}$$

But in general, when t represents the time,
p the acceleration,
x the space described,
v the velocity acquired, the following fundamental relations hold good for all variable motions, viz.:

$$dt = \frac{dx}{v}, p = \frac{dv}{dt} = \frac{dv.\,v}{dx}, \quad pdx = dv.\,v.$$

Consequently, if in order to allow of distinguishing the theoretical, vertically descending and ascending veins from each other, we substitute successively, in the last series of fundamental relations:

$$\text{for } t, -t_t, t_d, t_a,$$

$$\text{for } p, -p_t, p_d, p_a,$$

$$\text{for } v, -v_t, v_d, v_a,$$

$$\text{for } y, -y_t, y_d, y_a,$$

we will obtain:

1. For horizontal jets abstracted from the action of gravity outside of the reservoir (which for swift jets is very nearly the case for a length of trajectory equal to a a couple of diameters or so):

$$y_t = \frac{r\sqrt[4]{i'_{\binom{v}{a}}\, s + i_{\binom{v}{a}}\, x}}{\sqrt[4]{i_{\binom{v}{a}}\, s + x}} \tag{1_t}$$

$$v_t = \frac{\sqrt{2g\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) H \left(i'_{\binom{v}{a}}\, s + x\right)}}{\sqrt{i'_{\binom{v}{a}}\, s + i_{\binom{v}{a}}\, x}} \tag{2_t}$$

$$p_t = \frac{dv_t}{dx}\, v_t = g\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) H \left\{ \frac{1}{i'_{\binom{v}{a}}\, s + i_{\binom{v}{a}}\, x} - \frac{i_{\binom{v}{a}}\left(i'_{\binom{v}{a}}\, s + x\right)}{\left(i'_{\binom{v}{a}}\, s + i_{\binom{v}{a}}\, x\right)^2} \right\} \tag{3_t}$$

$$t_t = \int \frac{dx}{v_t} = \int \frac{dx\sqrt{i'_{\binom{v}{a}}\, s + i_{\binom{v}{a}}\, x}}{\sqrt{2g\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) H \left(i'_{\binom{v}{a}}\, s + x\right)}} \tag{4_t}$$

As all available experiments bearing on the subject, notably those recapitulated in Table X, seem to point to the fact that the mean value of the ratio $\frac{i_o}{i_c}$ of the respective alternating intensities of the moving force varies, with the absolute velocity of the water or the pressure in the reservoir and the area or radius of the cross-section of the vein, $i_{\binom{v}{a}}$, was introduced in the above equations to denote generally this mean ratio inside or outside of the reservoir between any two sections A O B and C E D and $i'_{\binom{v}{a}}$, to indicate the same mean ratio proper to the portion of vein lying within the reservoir between the plane of the orifice A O B and the plane of rest R S. (*See* Fig. 8.)

y_t is a minimum for $x = \infty$, when it becomes equal to $r\sqrt[4]{i_{\binom{v}{a}}}$.

y_t is a maximum for $x = -\, i'_{\binom{v}{a}}\, s$ when it becomes equal to ∞; v_t is a minimum for $x = -\, i'_{\binom{v}{a}}\, s$, when it becomes equal to o; v_t is a maximum for $x = \infty$, when the velocity becomes equal to:

$$\sqrt{\left\{2g\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) H\right\} \frac{1}{i_{\binom{v}{a}}}}$$

p_t is a minimum for $x = \infty$ when it becomes equal to o.

p is a maximum for $x = -\dfrac{i'_{\binom{v}{a}}\, s}{i_{\binom{v}{a}}}$ when it becomes equal to ∞.

$t_t = \infty$, both for $x = \infty$ and for $x = -\, i_{\binom{v}{a}}\, s$.

2° In vertically descending circular veins projected through simple horizontal orifices, where the acceleration p_d, is always equal to the acceleration p_t, of the theoretical horizontal vein, plus the acceleration g, produced by the never-ceasing force of gravity, in addition to that due to the hydraulic pressure stored in the reservoir, we have —

$$p_d = p_t + g = \left\{\left(\frac{1}{\underset{\binom{v}{a}}{i'}\, s + \underset{\binom{v}{a}}{i}\, x} - \frac{\underset{\binom{v}{a}}{i}\left(\underset{\binom{v}{a}}{i}\, s + x\right)}{\left(\underset{\binom{v}{a}}{i'}\, s + \underset{\binom{v}{a}}{i}\, x\right)^2}\right)\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) H + 1\right\} g \qquad (1_d)$$

$$\int p_d\, dx = \int\left(p_t + g\right) dx = \int\left(dv_t\, v_t + g\right) dx = \tfrac{1}{2} v_t^2 + gx = \int dv_d\, v_d = \tfrac{1}{2} v_d^2$$

whence—

$$v_d = \sqrt{v_t^2 + 2gx} = \sqrt{\frac{2g\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) H\left(\underset{\binom{v}{a}}{i'}\, s + x\right)}{\underset{\binom{v}{a}}{i'}\, s + \underset{\binom{v}{a}}{i}\, x} + 2gx} \qquad (2_d)$$

$$y_d = \frac{r\sqrt[4]{\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) H}}{\sqrt[4]{\frac{\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) H\left(\underset{\binom{v}{a}}{i'}\, s + x\right)}{\underset{\binom{v}{a}}{i'}\, s + \underset{\binom{v}{a}}{i}\, x} + x}} \qquad (3_d)$$

$$t_d = \int \frac{dx}{v_d} = \int \frac{dx}{\sqrt{\frac{2g\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) H\left(\underset{\binom{v}{a}}{i'}\, s + x\right)}{\underset{\binom{v}{a}}{i'}\, s + \underset{\binom{v}{a}}{i}\, x} + gx}} \qquad (4_d)$$

y_d is a minimum for $x = \infty$, where the radius of the vein becomes infinitely small, theoretically speaking.

y_d is a maximum for:

$$\frac{2g\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right)\left(\underset{\binom{v}{a}}{i'}\, s + x\right)}{\underset{\binom{v}{a}}{i'}\, s + \underset{\binom{v}{a}}{i}\, x} + 2gx = o \qquad (5_d)$$

viz: for—

$$x = \sqrt[2]{-H\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right)\left(\frac{\underset{\binom{v}{a}}{i'}}{\underset{\binom{v}{a}}{i}}\right) s + \frac{1}{4}\left(\frac{H\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) + \underset{\binom{v}{a}}{i'}\, s}{\underset{\binom{v}{a}}{i}}\right)^2} - \frac{1}{2}\left(\frac{H\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) + \underset{\binom{v}{a}}{i'}\, s}{\underset{\binom{v}{a}}{i}}\right) \qquad (6_d)$$

when the ordinate y_d, becomes infinitely great, the velocity v_d, being a minimum and equal to o.

v_d is a maximum for $x = \infty$, being then also infinitely great, theoretically speaking.

3° In the vertically ascending vein, where the retarding effect constantly produced by the force of gravitation is on the contrary inflecting, the liquid filaments outward and diminishing their previous inward inflection towards the axis—

$$p_a = p_t - g = \left\{\left(\frac{1}{\underset{\binom{v}{a}}{i'}\, s + \underset{\binom{v}{a}}{i}\, x} - \frac{\underset{\binom{v}{a}}{i}\left(\underset{\binom{v}{a}}{i'}\, s + x\right)}{\left(\underset{\binom{v}{a}}{i'}\, s + \underset{\binom{v}{a}}{i}\, x\right)^2}\right)\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) H - 1\right\} g \qquad (1_a)$$

$$\int p_a\, dx = \int (p_t - g)\, dx = \int (dv_t\, v_t - g)\, dx = \tfrac{1}{2} v_t^2 - gx = \int dv_a\, v_a = \tfrac{1}{2} v_a^2$$

whence—

$$v_a = \sqrt[2]{v_i^2 - 2gx} = \sqrt[2]{\frac{2g\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) H\left(i'_{\binom{v}{a}}\, s + x\right) - 2gx}{i'_{\binom{v}{a}}\, s + i_{\binom{v}{a}}\, x}} \qquad (2_a)$$

$$y_a = \frac{r\sqrt[4]{\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) H}}{\sqrt[4]{\frac{\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) H\left(i_{\binom{v}{a}}\, s + x\right) - x}{i'_{\binom{v}{a}}\, s + i_{\binom{v}{a}}\, x}}} \qquad (3_a)$$

$$t_a = \int \frac{dx}{v_a} = \int \frac{dx}{\sqrt[2]{\frac{2g\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) H\left(i'_{\binom{v}{a}}\, s + x\right) - 2gx}{i'_{\binom{v}{a}}\, s + i_{\binom{v}{a}}\, x}}} \qquad (4_a)$$

y_a is a maximum, when—

$$v_a^2 = 2g\left\{\frac{\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) H\left(i'_{\binom{v}{a}}\, s + x\right)}{i'_{\binom{v}{a}}\, s + i_{\binom{v}{a}}\, x} - x\right\} = o, \qquad (5_a)$$

viz: when—

$$x = \sqrt[2]{\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) H\left(\frac{i'_{\binom{v}{a}}}{i_{\binom{v}{a}}}\right) s + \tfrac{1}{4}\left(\frac{i'_{\binom{v}{a}}\, s - \left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) H}{i_{\binom{v}{a}}}\right)^2} - \tfrac{1}{2}\left\{\frac{i'_{\binom{v}{a}}\, s - \left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) H}{i_{\binom{v}{a}}}\right\} \qquad (6_a)$$

Again, y_a is a maximum and at the same time v_a a minimum, when—

$$d\left\{\frac{i'_{\binom{v}{a}}\, s + i_{\binom{v}{a}}\, x}{\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) H\left(i'_{\binom{v}{a}}\, s + x\right) - i'_{\binom{v}{a}}\, sx - i_{\binom{v}{a}}\, x^2}\right\} = o \qquad (7_a)$$

whence—

$$x = \pm\sqrt[2]{-\frac{i'_{\binom{v}{a}}}{i_{\binom{v}{a}}} Hs + \frac{i'_{\binom{v}{a}}}{i^2_{\binom{v}{a}}}\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right) Hs} - \frac{i'_{\binom{v}{a}}}{i_{\binom{v}{a}}}\, s = \pm\sqrt{Hs\left\{\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right)\frac{i'_{\binom{v}{a}}}{i^2_{\binom{v}{a}}} - \frac{i'_{\binom{v}{a}}}{i_{\binom{v}{a}}}\right\}} - \frac{i'_{\binom{v}{a}}\, s}{i_{\binom{v}{a}}} \qquad (8_a)$$

All the experiments made bearing on the question of viscosity and mutual interference combined, seem to point to the conclusion that the loss of velocity head, caused by this complex resistance increases, in some measure, with the head, and diminishes as the area of the orifice or cross-section of the vein increases, but in obedience to what precise laws the variations of the coefficients $c\binom{\text{vel}}{\text{head}}$ and i take place, is not easy to establish from the experimental data on record.

Outside of the reservoir, the fluid molecules are not directly subjected to pressure, comparatively to what takes place inside; but the resistance of the air has also to be taken into account. Horizontal jets produced under heads varying from 1 foot upwards, with circular orifices, varying, say, from 1 to 7 inches in diameter, are said to reach, according to all authorities on the subject, which have come into my hands, to the end of the same distance measured from the orifice, as if the greatest

velocity of the jet at or near this orifice was the same as that acquired by a heavy body after falling freely through a space equal to the mean height of the water surface in the reservoir above the opening in its side. It does not yet appear to be absolutely established, that the horizontal projections of jets formed in circular orifices, which are pierced in thin plates, invariably coincide with those of a solid body having a velocity equal to $\sqrt{2gH}$.

According to Weisbach, the coefficients of velocity increase with the heads and Michelotti's experiments on horizontal jets go to show, on the contrary, that they diminish as the heads increase; thus, while for a head of $7\frac{1}{2}$ feet the coefficient of velocity was found by the latter to be 993, for a head of $23\frac{1}{2}$ feet, it was only 983, with the same orifice.

This matter is still involved in much uncertainty and must remain so until some philanthropically disposed Government, wealthy corporation, rich nobleman or merchant prince may choose to take sufficient interest in the advancement of hydraulic science, to set apart the funds required for making conscientious and systematic collections of all reliable experimental data having a bearing on this subject, which are to be found in existing works and archives, and to organize a proper hydraulic service, amply provided with all the necessary apparatuses and appliances, for the purpose of filling, with the results of fresh experiments, the numerous gaps which must inevitably be found to declare themselves after the work of compilation is completed and for verifying such results of old experiments as might appear to be of a doubtful character.

The following table (XIII.) shows the values of $\left(\begin{smallmatrix}\text{coeff.}\\ \text{vel.}\\ \text{orif.}\end{smallmatrix}\right)$ for efflux in air, which were arrived at by different experimenters, with various orifices and heads, and also the corresponding values of $\left(\begin{smallmatrix}\text{coeff}\\ \text{vel.}\\ \text{heads.}\\ \text{orif.}\end{smallmatrix}\right)$ the coefficient of velocity head of efflux in the plane of a circular orifice in a thin plate.

TABLE XIII.

1	2	3	4	5	6	
No.	Diameter of orifice in inches.	H Head of water in reservoir above centre of orifice.	$\left(\begin{smallmatrix}\text{coeff.}\\ \text{vel.}\\ \text{orif.}\end{smallmatrix}\right)$ Coefficient of velocity of efflux in plane of orifice.	$\left(\begin{smallmatrix}\text{coeff.}\\ \text{vel}\\ \text{head orif.}\end{smallmatrix}\right)=\left(\begin{smallmatrix}\text{coeff.}\\ \text{vel.}\\ \text{orif.}\end{smallmatrix}\right)^2$ Coefficient of velocity head of efflux in plane of orifice.	Authority.	Remarks.
1	0·15945	0·8817 inches.	0·6628	0·4393	Weisbach	
2	0·2000	14 "	0·623	0·3881	Dr. Matthew Young..	
3	0·384	51 "	0·6210	0·3856	Steckel	Orifice in bottom of reservoir—Jet vertically descending.
4	"	44 "	0·6263	0·3922	"	" " "
5	"	35 "	0·6259	0·3917	"	" " "
6	"	29 "	0·6277	0 3940	"	" " "
7	"	19 "	0 6268	0·3929	"	" " "
8	"	12·10 "	0·6281	0·3945	"	" " "
9	"	3·06 "	0·6544	0·4282	"	" " "
10	0·394	339·839 feet.	0·5964	0 3557	Weisbach	
11	"	44·536 "	0·6257	0·3915	"	
12	"	35·786 inches.	0·6218	0.3866	"	
13	"	2·133 feet.	0·6730	0·4529	Castel.....................	
14	"	23·621 inches.	0·6092	0·3711	Weisbach................	
15	"	1·017 feet.	0·6540	0·4277	Castel.....................	
16	"	9·842 inches.	0·6179	0·3818	Weisbach	
17	"	3·937 "	0·6368	0·4055	"	
18	"	0·787 "	0·6400	0·4096	"	
19	0·3996	33·7849 "	0·6416	0·4117	Venturi	Orifice in top of closed reservoir—Ascending jet a little declined from the vertical.
20	"	0·8525 "	0·6556	0·4298	Weisbach	
21	0·3996	59 683 "	0 6319	0·3993	Venturi	" " "
22	0·400	3·100 "	0·6662	0·4438	Steckel.....................	Orifice in bottom of reservoir—Stream or vein vertically descending.
23	"	2 970 "	0·6726	0·4524	"	" " "
24	"	2·920 "	0·6727	0·4525	"	" " "
25	"	2·850 "	0 6743	0·4547	"	" " "
26	0·4185	3·030 "	0·6802	0·4627	"	" " "
27	0 420	3·070 "	0·6775	0·4590	"	" " "
28	0·482	3·000 "	0·6803	0·4628	"	" " "
29	0·484	2 810 "	0·6844	0·4684	"	" " "

TABLE XIII—*Concluded.*

1	2	3	4	5	6	
No.	Diameter of orifice in inches.	H Head of water in reservoir above centre of orifice.	$\left(\begin{smallmatrix}\text{coeff.}\\\text{vel.}\\\text{orif.}\end{smallmatrix}\right)$ Coefficient of velocity of efflux in plane of orifice.	$\left(\begin{smallmatrix}\text{coeff.}\\\text{vel.}\\\text{head orif.}\end{smallmatrix}\right) = \left(\begin{smallmatrix}\text{coeff.}\\\text{vel.}\\\text{orif.}\end{smallmatrix}\right)^2$ Coefficient of velocity head of efflux in plane of orifice.	Authority.	Remarks.
30	0·533	4·263 feet.	0·616	0·3795	Bossut	Orifice in side of reservoir.
31	0 533	9 600 "	0·613	0·3758	"	" "
32	0·590	0·453 "	0 632	0·3994	"	" "
33	0·590	0·984 "	0·617	0·3807	"	" "
34	1·027	2·372 "	0·618	0·3819	Eytelwein	
35	1·066	9·600 "	0·617	0·3807	Bossut	Orifice in side of reservoir.
36	1·066	7·327 "	0·619	0·3832	Michelotti	" "
37	"	4·263 "	0·616	0·3795	Bossut	" "
38	"	0·6217 inches.	0 649	0·4212	"	" "
39	1·181	2·676 "	0·629	0 3956	Castel	
40	1·614	2·887 feet.	0·622	0 3869	Venturi	Orifice in side of reservoir.
41	1·599	3·553 "	0 605	0·3660	"	" "
42	2·126	7·218 "	0·607	0·3684	Michelotti	" "
43	2·132	23·344 "	0·605	0·3660	"	" "
44	2·132	12·493 "	0 605	0·3660	"	" "
45	3·189	22·179 "	0·597	0·3564	"	" "
46	3·189	12·500 "	0.612	0·3745	"	" "
47	3·189	7·349 "	0 613	0·3758	"	" "
48	6·378	12·008	0 619	0·3832	"	" "
49	6·378	6·923	0·619	0 3832	"	" "

The following series of coefficients for circular orifices from Rennie's experiments with orifices from $\frac{1}{4}$ inch to 1 inch diameter

The following series of coefficients for circular orifices from Rennie's experiments with orifices from ¼ inch to 1 inch diameter under heads from 1 foot to 4 feet extracted from Mr. Neville's work, I have purposely given separately from those entered in Table XIII, as it tends to prove, apparently contrary to the experience of other experimenters, including myself, that the coefficients of efflux or velocity in the orifice increase not only as the depths decrease but also simultaneously as the areas of the orifices are diminished:—

TABLE XIV.

No.	Diameter of orifices in inches.	H. Head of water in reservoir above centre of orifice.	$\left(\text{coeff. vel. orif.}\right)$ Coefficient of velocity of efflux in plane of orifice.	$\left(\text{coeff. vel. head orif.}\right) = \left(\text{coeff. vel. orif.}\right)^2$ Coefficient of velocity head of efflux in plane of orifice.	Authority.	Remarks.
1	0·25	1 foot.	0·671	0·4502	Rennie.	Orifice in side of reservoir.
2	"	2 feet.	0·653	0·4264	"	" "
3	"	3 "	0·660	0·4356	"	" "
4	"	4 "	0·662	0·4382	"	" "
5	0·50	1 foot.	0·634	0·4020	"	" "
6	"	2 feet.	0·621	0·3856	"	" "
7	"	3 "	0·636	0·4045	"	" "
8	"	4 "	0·626	0·3919	"	" "
9	0·75	1 foot.	0·644	0·4147	"	" "
10	"	2 feet.	0·652	0·4251	"	" "
11	"	3 "	0·632	0·3994	"	" "
12	"	4 "	0·614	0·3770	"	" "
13	1·00	1 foot.	0·633	0·4007	"	" "
14	"	2 feet.	0·619	0·3832	"	" "
15	"	3 "	0·628	0·3944	"	" "
16	"	4 "	0·584	0·3411	"	" "

Mr. John Neville says, at page 55 of the 3rd edition of his work:

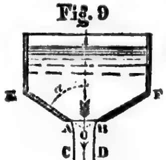
"It may be remarked, in passing, how universal the coefficients ·613 to ·628 are for all forms of orifices in thin plates; or with the outside arrises chamfered. Indeed, the coefficient ·62 may always be used with certainty for practical purposes, for every orifice of this kind (round or square), whether at the surface, in the form of a notch, or at the sides or bottom of a vessel, if the section of the approaching water be large in proportion to the area of the discharging orifice or notch. By coefficient, of course, is here meant that decimal which, multiplied by the theoretical value, gives the practical result; and this is substantially the same for notches and orifices sunk below the surface."

It is evident, judging by the coefficients given in Tables XIII. and XIV., that the case is quite different as regards theoretical computations.

All the arguments advanced thus far in support of the theoretical formation of the *vend contractà*, as above, are based on the teachings of phenomena pertaining to veins generated through circular orifices in thin, perfectly flat plates. Notwithstanding this, it is readily perceived, upon reflection, that no reason exists why the principles deduced from the enquiries instituted should not also hold good for veins projected through all kinds of circular orifices, viz., whether efflux takes place through a plane at right angles to the direction of motion or through an interior, cylindrical, divergent or convergent tube, without touching the sides.

That there is something abnormal in connection with this feature of the theory proposed, which requires to be looked into and cleared up, appears from the following considerations:

Fig. 9



It is well known that when the axis I X of a stream A B C D makes an acute angle E O I or a with the wall E A O B F, as in Fig. 9, the contraction is smaller, and when the axis I X makes an obtuse angle E O I or a_2, with the wall E A O B F, as in Fig. 11, it is greater than in the case of a vein projected through an orifice A O B pierced in a flat plate E A O B F, where the angle E O I $= a_1$ is a right angle, as shown in Fig. 10.

Fig. 10

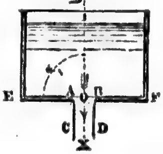


Fig. 11

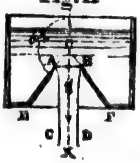
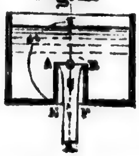


Fig. 12



Borda, Bidone and Weisbach have found that when the angle E O I$=a_3$ in fig. 12, reaches 180°, the coefficient of contraction is reduced to a mean value of 0·53—and in two of his experiments Bidone obtained coefficients as low as 0·50 nearly.

Dr. Weisbach made a series of experiments with a great number of mouth-pieces, 2 centimeters or 0·787 inch wide, and under pressures varying from 1 to 10 feet; the results of his experiments with respect to efflux, were as follows:

Angle E O I.	180°	157½°	135°	112½°	90°	67½°	45°	22½°	11¼°	5⅝°	0°
Coefficient of efflux.	0·541	0·546	0·577	0·606	0·632	0·684	0·753	0·882	0·924	0·949	0·966

As a small loss of velocity always takes place during efflux, he estimates that the coefficients of contraction are from 1 to 2 per cent. greater than the coefficients of efflux. Under a head of ~~about~~ 2·475 inches I found that the coefficient of efflux through an orifice of 0·416 inch in diameter, with a sharp edge in a wall whose sides were inclined at an angle of 157½° to the axis of the vein, was as high as 0·598 instead of only 0·546. Furthermore, under a head of 2·65 inches, the coefficient of efflux obtained by me for a jet formed in an orifice 0·405 inch diameter, pierced in a wall inclined to the axis of the stream at an angle of 135° was as high as .657 when the aperture, instead of having a sharp edge, was surrounded by a flat rim about $\frac{1}{68}$ of an inch wide in the plane of this orifice. (See Table XI½, page 31).

For the present, however, it is not necessary to attach much importance to these comparatively small variations in the coefficients of efflux and contraction; the broad fact remains, that both the coefficients of efflux in the orifice and coefficients of contraction are variable with the degree of inclination of the sides of the truncated cone A B F E, whose small base A B, constitutes the orifice, to the direction followed by the current or axis of the stream.

The fluctuations of these coefficients are due, as several experimenters have remarked, to the fact of the molecules which flow towards the orifices having to suffer various deviations from the initial directions followed by them while finding their way through the orifice, to form the corresponding vein in each case.

In this respect, viz.: as regards deviation from the directions to be followed in order that the maximum amount of *vis viva* may be produced which may be designated as the normal direction—the molecules flowing through a circular orifice in a thin flat plate—are clearly not an exception to the general rule. That is to say, some of the molecules which are between the plane of rest R K S, and the plane of the aperture A O B (Fig 8) and particularty those lying nearest to this latter plane, must necessarily be deviated to a small extent from the normal direction just described, and it is evident also that, in its passage from the reservoir outward, through an orifice in a thin plate, the liquid stream is not strictly confined, inside of the reservoir, within a truncated conoïd resembling that which is generated by the revolution of the curve determined by equation (1_t) on its longtitudional axis X E, Fig 8.

It will be observed that even in this, the simplest kind of orifice, the free efflux of the liquid is somewhat interfered with, and friction against the metallic envelope being abstracted, the velocity in the plane of the orifice must be slightly smaller and contraction outside of the reservoir correspondingly greater than if the flow had taken place through a conoïdal mouthpiece, so proportioned that within it motion would diminish gradually—proceding from the plane of the orifice to the plane of rest—solely by virtue of the continuous increase of the field of action, in accordance with some fixed law, toward the interior of the reservoir perpendicularly

to the plane of the orifice, as compared to the motion imparted by the original impulse to the first elementary layer or sheet of liquid which leaves this plane on the aperture being opened.

It is therefore evident that even if water was devoid of viscosity and if absolutely no resistance was encountered in the passage through the atmosphere, nor friction of any kind generated, a vein projected through a circular orifice in a thin plate with a sharp edge, under a constant head or pressure, could yet not be called a theoretically perfect fluid jet, to wit: a jet composed of a succession of elementary fluid sheets, detached from the body of liquid contained in a state of rest within the reservoir, with gradually increasing velocities and free from all lateral disturbance by extraneous contiguous molecules.

The head K X (Fig. 8), the cross-section C D, and its distance K E, from the origin of motion or plane of rest R S within reservoir, being given, the corresponding perfect fluid circular vein may be defined to be the stream possessing the greatest possible amount of energy to be obtained under the conditions imposed, at the given cross-section as well as at the section of maximum contraction.

Now, a stream or vein flowing through an orifice in a thin plate, under a comparitively small head of say, 5 or 6 diameters or thereabouts, cannot differ sensibly from the theoretically perfect conoidal stream just defined—more especially the portion outside of the reservoir hence the coefficients of velocity of efflux and contraction corresponding to such an orifice—viz., the ratio between the actual velocity of the liquid in the orifice and that due to the head must coincide very nearly with the theoretical coefficients of velocity of efflux and contraction corresponding to a maximum production of living force and may, therefore, be taken as the measure of these latter, very little error being made.

Again, we have already seen that the largest coefficient of velocity of efflux, in air obtained with an orifice in a thin plate, is about 0·70; this figure (or say $\sqrt{\frac{1}{2}}$ with Newton) may, therefore, be considered to be the true value (nearly) of the coefficient of velocity of efflux of the corresponding perfect theoretical vein, viz.: it may be considered that one-half of the head of water in any reservoir is essentially consumed or utilized in ejecting liquid through a simple orifice, and the other half in generating additional velocity or *vis-viva.*

Finally, by adhering to the principle verified by experiment, within certain limits at least, that the energy developed is proportional to the head or pressure in the reservoir, the probable theoretical coefficient of maximum contraction of a naturally contracted vein composed of perfectly fluid matter, in which case no loss whatever of force could take place, is thus found to be equal to $\sqrt[4]{\frac{1}{2}} = 0{\cdot}8408$, not at a distance equal to the radius of the orifice, or so, from the reservoir; but at an infinite distance from the same.

APPLICATIONS OF THE NEW THEORY.

COMPARISON OF THEORETICAL COMPUTATIONS WITH EXPERIMENTAL RESULTS.

After constructing the fundamental formula required to determine theoretically the motions, forms, &c., of the most elementary kinds of circular contracted liquid veins that are formed through an orifice in a thin plate, I will now attempt to employ some of these equations in the numerical computation of quantities and dimensions, previously established by means of actual measurements of veins of water produced in nature, and of the corresponding discharges in a fixed length of time.

In this manner, I may perhaps succeed in removing some of the ground for hesitation, respecting the acceptance of the hydraulic theory presented above, which the want of concordance of theoretical with experimental results has not, without good cause, proved to be in many similar instances.

Distrust as regards the soundness of the hydraulic theory here presented, would be the more natural, as I found the use of complex and comparatively obscure phraseology unavoidable when endeavoring to describe the effects produced on an infinite number of molecules of matter, liable to change their relative positions at every instant—by an agent, whose action is not directly perceptible to the

touch nor measurable, such as proves to be the force which holds together the constituent elementary particles of every mass of liquid, the reality of whose influence is apparently incapable of being rendered manifest to our senses in any other manner than through the variations of form and pressure brought about by it in various kinds of liquid veins and moving fluid bodies.

On account of the limited number of reliable experimental data of the proper kind that are available at present, it is not to be expected that I should be able to furnish numerous examples of successful applications of the fundamental equations above laid down, to the determination of the forms and other properties of all the different kinds of fluid veins to be met with in nature, as well as of the discharges from tubes, pipes, &c. Indeed, I was forced, in nearly all the cases exemplified, to content myself with computing mere rough approximations to the quantities and dimensions sought; but although rough, the results will be found to be indicative of the soundness of the principles of the new theory.

HORIZONTAL JETS.

The first experiment which I have chosen in this connection, for comparison with theory, is one of a truly original and scientific character. We owe it to the initiative of Mr. T. Trudeau, the present Deputy Minister of the Department of Railways and Canals, of Canada, so justly distinguished for his learning and scientific attainments, who is for ever taking the greatest interest in the advancement of those branches of the natural sciences, which are more especially connected with the duties of the important office which he so ably fills.

In order to obtain an infallibly correct representation of the form assumed by the contracted vein at its exit from the reservoir, Mr. Trudeau conceived the happy idea of having a photographic view taken of a liquid vein projected horizontally through a circular orifice A B, [illegible], plate II, 0·530 inch in diameter, under a constant head or pressure of about 14 inches.

This orifice was pierced, on the lathe, in a polished brass plate C D $\frac{1}{16}$ inch thick, being flaired out from 0·530 inch in diameter at A B, on the outer face, to about 4 inches in diameter at C D, on the face within the reservoir, so as to form a conoidal cavity resembling, as near as could be judged by a close inspection of the outflowing fillets, to the inner portion of a contracted liquid vein projected, under an equal head of water, through a circular orifice in a thin plate, having about the same diameter. By this arrangement it was possible to photograph a far greater length of the more important portion of the vein, than if the orifice had been pierced in a thin plate reduced to a feather edge, from the outer towards the inner face, viz., that in contact with the water.

It will also be noticed that, formed in these conditions, the vein outside of the reservoir must have presented a profile differing less from that of the true theoretical fluid vein referred to at page 48, than under any other circumstances, and the contraction must undoubtedly have proved smaller than in the case of a corresponding vein projected under the same head through an orifice in a thin flat plate.

On the other hand, this mode of proceeding gave rise to some uncertainty as to the precise location of the origin of the nearly theoretically perfect fluid vein thus obtained, and therefore, also, with respect to the exact diameter of the cavity in the plate corresponding to this origin or, more properly, the plane where the velocities due to the forces $f i_o$ and $f i_c$ are equivalent. This difficulty was got over, however, by fixing the value of the coefficient of contraction, viz.: $c_c = \iota^{\varkappa}$, approximately at 0·83—at a distance of about one diameter from the orifice—(this number being the mean value of the coefficient of maximum contraction of a vein projected through an orifice 0·482 inch diameter, under a head of 3 inches, found by direct measurement, See Table IV), on the ground that the contraction of a vein produced under a head so small in comparison to its diameter, must also have proved nearly the same as the corresponding contraction in a theoretically perfect fluid vein, viz.: one unaffected by either friction, or resistance of the atmosphere, and otherwise undisturbed in its natural forward movement.

From the negative obtained, which was much smaller than the natural size of the vein, enlarged views were made in a solar camera, the actual diameter of the vein being in this manner augmented from 0·50 inch to 8·36 inches. These pictures were skilfully executed by Mr. S. McLaughlin, the experienced photographer of the above named Department, so that an outline of figure, sufficiently clear and sharp, was obtained, to allow of accurately measuring, by scale, the coordinates of the curve forming the longitudinal profile of the vein under consideration, for a distance of about $\frac{2}{3}$ of an inch or $1\frac{1}{2}$ diameters from the plane of the orifice. A *fac simile* of this profile, together with an approximate enlarged section, ~~[illegible]~~ of the brass plate, is given in Plate II; and Table XV, which here follows, shows the lengths of the ordinates computed by means of equation (1_t) side by side with those measured on the photographic record.

TABLE XV.

x_t, Abciss from origin O in centre of orifice in thin plate — Inches.	y_t, Ordinate perpendicular to axis of vein, measured on photographic record. — Designation on plate II.	y_t, Ordinate perpendicular to axis of vein, measured on photographic record. — Inches.	y_t, Ordinate perpendicular to axis of vein, computed by formula (1_t) — Inches.	Notes.
−0 9893	....	....	∞	
−0·7500	....	....	5·7921	
−0·5000	....	....	5·0163	
0·0000	$\overline{\text{OA}}$	....	4 4578	
+0·3380	$\overline{\text{·338a}}$	4·2800	4 2799	
+0 5000	5b	4·2100	4·2183	
+1·0000	$\overline{\text{1c}}$	4·0600	4 0822	
+1·5000	$\overline{\text{1·5d}}$	3·9700	3 9938	
+2·0000	$\overline{\text{2e}}$	3 9000	3·9315	
+3 0000	$\overline{\text{3f}}$	3·8200	3·8494	
+4·0000	$\overline{\text{4g}}$	3 7650	3·7976	
+5·0000	$\overline{\text{5h}}$	3 7450	3·7618	
+6·0000	$\overline{\text{6I}}$	3·7250	3·7357	
+7 0000	$\overline{\text{7J}}$	3·7100	3 7157	
+7·5000	$\overline{\text{7·5K}}$	3·7050	3·7080	
+8 0000	$\overline{\text{8l}}$	3·7050	3·7000	
+9·0000	$\overline{\text{9m}}$	3·7100	3 6872	
+10·0000	$\overline{\text{10n}}$	3·7150	3·6767	
+11·0000	$\overline{\text{11o}}$	3 7170	....	
+12·0000	$\overline{\text{12p}}$	3·7220	....	
+13·0000	$\overline{\text{13Q}}$	3·7250	....	
∞	....	....	3·5662	

Notes.

For $i^{1/4}_{(\frac{v.}{a.})} =$ ·80 on an average between the points O and E or 8, Plate II, $i_{(\frac{v.}{a.})} =$ ·4096. Also for $c_{\text{cont.}} =$ ·83, $r_{\text{orif.}} =$ 4·4578 inches whence $\frac{0·53 + 4·4578}{4·2799} =$ 0 55 in. = natural size of $r_{\text{orif.}}$ Hence, substituting the numerical values for the symbols, we have, at the distance of 8 inches from the plane of the orifice where the diameter was found to be a minimum, and equal to 3·70 inches by measurement:

$$\frac{r_{\text{orif.}}\sqrt[4]{i_{(\frac{v.}{a})}\, s_0 + i_{(\frac{v.}{a.})}\, x}}{\sqrt{i_{(\frac{v.}{a.})}\, s_0 + x}} = 3·70 \text{ inches,}$$

whence we deduce $s_0 =$ 2·4154 inches in the enlarged vein and $s_0 = \frac{2·4154}{4·4578} r_{\text{orif.}} =$ 0·5419 $r_{\text{orif.}} =$ 0·1495 inch in the natural vein of water; also $i_{(\frac{v.}{a.})}\, s_0 =$ ·4096 = ·5419 $r_{\text{orif.}} =$ ·22196 $r_{\text{orif.}} =$ ·06104 inch in this last vein, viz.: natural size. Thus s_0 stands for an auxiliary space over which a body solicited with a uniform acceleration equivalent to the mean acceleration generated by the force $f i_o$ outside of the reservoir, would have to travel within the reservoir, in order to attain at O a velocity equal to that generated by this variable force $f i_o$ within the reservoir during the passage of the liquid from N to O (Fig. 8.)

As $i_{\left(\frac{v}{a}\right)}$ appears to increase simultaneously with the velocity of the water in the vein, and nearly as the square root of this velocity, judging by the values of $i_{\left(\frac{v}{a}\right)}$ computed in the case of a vertically descending vein projected through an orifice 0·4 inch in diameter, which are given hereafter at page 52; $(\cdot 80)^4 = \cdot 4096$ was assumed to be the approximate mean value of this ratio, $i_{\left(\frac{v}{a}\right)}$ along the portion O E, or $\overline{os}$ of Plate II the natural vein under consideration, instead of $c_c{}^4 = (\cdot 83)^4 = \cdot 4747$, which is more properly the particular value corresponding to the section O E D.

The distance O N $= s$ (Fig. 8) of the plane of rest P Q, from the plane of the orifice A O E not having been ascertained by direct measurement, as was done by me for the vertically descending vein (See experiments j, Tables VII and VIII), for the very good reason that when the experiment under consideration was made, there was no apparent object in establishing the position of this plane with accuracy; the length of an appropriate auxiliary space $s_0 = 0{\cdot}14956$ inch, equivalent, as regards generation of motion (when $i_{\left(\frac{v}{a}\right)}$ is constant) to the actual length of O N $= s$ in the reservoir, was established, as shown in the last table, No. XV, in the column headed "Notes," on the supposition that the value of $i_{\left(\frac{v}{a}\right)}$, instead of diminishing, as we proceed from any point E towards the plane R S within the reservoir, and increasing when we travel in the opposite direction along the path of the vein—remains constantly equal, on an average, to 0·4096, along the portion A O B S K R of the vein which lies within the reservoir, the same as for the portion on the outside.

Along this inner portion of the naturally contracted vein the actual mean value of $i_{\left(\frac{v}{a}\right)}$ is probably, as just pointed out, less than 0·4096, decreasing possibly from, say 0·41, on an average, within the space of one diameter or so outside the reservoir in front of the plane of the orifice A O B to 0 at the plane P Q, corresponding to $x = s$, consequently the actual length of s must evidently exceed 0·14956 inch, say, in the ratio of 0·41 to 0·20, whence $s = 0{\cdot}30$ inch nearly; but the introduction of an auxiliary space s_0, while facilitating the work of computation, evidently, in no way affects or invalidates the final results.

It is, of course, not pretended that the values of c_c, $i_{\left(\frac{v}{a}\right)}$, s_0, r_{orif} determined in the manner just described, are correct, in a theoretical sense, more especially, as apart from other shortcomings, the action of gravity on the vein outside of the reservoir was neglected, the cavity in the brass plate undoubtedly different in a greater or less degree from the true form, and the resistance of the air had also necessarily to be left out of consideration. I think, however, that the close coincidence of the enlarged photographic record of the natural vein with the curve traced out on paper, by means of ordinates, computed with the aid of the formulas established, can reasonably be accepted as a fair indication of the soundness of the theory on which they are based.

The indications are that the mean values of $i_{\left(\frac{v}{a}\right)}$ vary approximately, in horizontal veins abstracted from gravity, as shown hereunder, viz.:

When	$x=0.$ (in the plane of the orifice),	$i=0{\cdot}87$	of the maximum value proper to the vein.		
"	$x=0{\cdot}1r$ orif.	$i=0.90$	"	"	"
"	$x=0.2r$ orif.	$i=0.925$	"	"	"
"	$x=0{\cdot}4r$ orif.	$i=0{\cdot}955$	"	"	"
"	$x=0{\cdot}6r$ orif.	$i=0{\cdot}97$	"	"	"
"	$x=0{\cdot}8r$ orif.	$i=0{\cdot}98$	"	"	"
"	$x=1{\cdot}0r$ orif.	$i=0{\cdot}99$	"	"	"
"	$x=1{\cdot}5r$ orif.	$i=0{\cdot}995$	"	"	"
"	$x=2{\cdot}0r$ orif.	$i=1{\cdot}000$	"	"	"

With regard to the precise form which the conoïdal cavity turned in the brass plate should have had, I do not see, on account of the interference with free

efflux, of the fluid particles drawn into the theoretically perfect conoïdal stream, between the orifice in a thin plate and the plane of rest R S, being a factor of disturbance of which it is impossible to form an estimate, that it can well be arrived at otherwise than by making repeated trials with mouth-pieces variously proportioned. There can be no doubt, however, but that the distance O K=0·9893 as determined in Table X V is slightly shorter than it should be.

If we took for granted that the law, according to which i apparently varies, is general, the conditions of such variation might possibly be directly combined with the other relations already established, and new equations more generally applicable to the class of veins under consideration could then be constructed.

Such a course would, however, tend to bury effectually out of sight, under what Mr. Trautwine has chosen to call mathematical rubbish, perhaps not altogether without some reason, fundamental principles which are, of their own nature, far from being easily discerned and understood, even when exposed and described in the fullest and clearest manner possible. I have, therefore preferred, not to attempt such algebraical combinations at present, contenting myself with introducing in the applications of these formulae which here follow such values of $i_{\binom{v}{a}}$ as would be required by the particular circumstances of the cases considered, keeping constantly in view that in general: the larger the head or pressure in comparison to the orifice, (1) the greater the value of $i_{\binom{v}{a}}$ in accordance with the law just enunciated, (2) the greater the protrusion of the vein from the orifice A O B, whence (3) the less the distance $s=\overline{ON}$ from the plane of the orifice to the plane P Q, where perfect equilibrium between the liquid particles ceases to be disturbed, whence also (4) the smaller the coefficient of the velocity head of efflux $\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\end{smallmatrix}\right)$ through an orifice in a thin plate in comparison to unity, which is that of the velocity due to the fall of a heavy body through a space equal to the total head of water in the reservoir above the orifice.

VERTICALLY DESCENDING VEINS.

The new theory was applied as follows to the determination of the value of $i_{\binom{v}{a}}$ at several points of the vertically descending circular vein projected under a head H=2·99 inches through an orifice in a thin plate 0·4 inch diameter, which I measured with points mounted on a diaphragm, as already described, the dimensions used being those given in Table III.

The numerical value given to $i_{\binom{v}{a}}\, s$, which represents the distance between the plane of the orifice and the plane of rest within the reservoir, is that which was determined experimentally, as explained, by introducing a cylindrical pin or rod 0·185 inch diameter into the reservoir, from above, opposite the orifice, approaching its base by means of screw motion, towards the plane of that orifice and establishing the lowest or limiting position of the base of the rod for which the volume of water discharged in the unit of time remained a maximum with a constant head—the cylinder being raised a small distance at a time and the corresponding discharge measured in every position. As this limit was reached approximately when the base of the cylindrical rod stood 0·24 to 0·25 inch above and back of the plane of the 0·4-inch circular opening in the thin plate, I put, accordingly, $i_{\binom{v}{a}}\, s=0{\cdot}25$ inch.

Substituting, therefore, in the following expression for $i_{\binom{v}{a}}$ in terms of y, x, H, r $i'_{\binom{v}{a}}$s and $\left(\begin{smallmatrix}\text{coeff.}\\ \text{vel.}\\ \text{head.}\\ \text{orif.}\end{smallmatrix}\right)$, which is deduced directly from equation (3_d) viz:

$$i_{\binom{v}{a}} = \frac{r^4 \left(\begin{smallmatrix}\text{coeff.}\\ \text{vel.}\\ \text{head.}\\ \text{orif.}\end{smallmatrix}\right) H\, i'_{\binom{v.}{a.}}\, s - y^4 \left(\begin{smallmatrix}\text{coeff.}\\ \text{vel.}\\ \text{head.}\\ \text{orif.}\end{smallmatrix}\right) H\, i'_{\binom{v.}{a.}}\, s + \left(\begin{smallmatrix}\text{coeff.}\\ \text{vel.}\\ \text{head.}\\ \text{orif.}\end{smallmatrix}\right) H\, x + x\, i'_{\binom{v.}{a.}}\, s}{x^2 y^4 - x\, r^4 \left(\begin{smallmatrix}\text{coeff.}\\ \text{vel.}\\ \text{head.}\\ \text{orif.}\end{smallmatrix}\right) H} \qquad (9)$$

2·99 inches for H, 0·25 inch for $i'_{\left(\substack{v.\\a.}\right)}s$, 0·2 inch for r, 0·44382 for $\left(\substack{\text{coeff,}\\\text{vel,}\\\text{head,}\\\text{orif,}}\right)$ as found in Table XIII, and for the coördinates y and x, successively, the dimensions obtained by direct measurement, as given in Table III, we find—

TABLE XVI.

x_d Abcissa measured from plane of orifice in thin plate downward.	y_d Ordinate.	$i_{\left(\substack{v.\\a.}\right)}$ —	Remarks.
1·000	0·1515	0·29737	
1·535	0·1480	0·37099	
2·535	0·1415	0·42937	These two values of $i_{\left(\substack{v.\\a.}\right)}$ do not seem to be in harmony with the others. It may be remarked, however, that a very slight error in the measurement of the diameter affects the value of $i_{\left(\substack{v.\\a.}\right)}$ considerably.
5·535	0·1240	0·35735	
10·535	0·1120	0·43550	
15·535	0·1035	0·43807	

These results seem to indicate that $i_{\left(\substack{v.\\a.}\right)}$ increases simultaneously with the velocity, and nearly as the square root of this velocity. Moreover, that for a mean diameter of about ½-inch and a velocity of say 120 inches or 10 feet per second $i_{\left(\substack{v.\\a.}\right)} = 0{\cdot}44$ nearly, in a vein projected through an orifice in a thin plate. A portion of the differences obtaining between the values of $i_{\left(\substack{v.\\a.}\right)}$ at various depths is, however, due to the fact of the plane of the theoretical orifice not being coincident with that of the orifice in the thin plate.

It is not usual to find that restrictions are made by authors on hydraulics respecting the uniformity of the discharging power of an orifice pierced in a thin plate; taking into account the position of its plane in relation to the horizon and the direction of the stream. No doubt, practically speaking, under the same head, the discharge through an orifice in a thin plate remains constant, whether this orifice lies in a horizontal, vertical, or any plane inclined to the horizon or vertical. From a theoretical standpoint, however, I am inclined to believe that the discharge through such on orifice, the head being constant, must be slightly greater for a vertically discending vein, especially under small heads, than it would be if the liquid stream followed a horizontal direction at its exit from the reservoir, notwithstanding the increased converganco and consequent mutual interference of the fillets in the immediate vicinity of the plane of the orifice outside of the reservoir, which are due to the additional acceleration suddenly imparted to the fluid particles by the action of gravity.

VERTICALLY ASCENDING JETS.

Dr. Weisbach gives, in his admirable treaties of Mechanics*, the following table where the heights reached by vertically ascending jets projected through orifices in

*Page 880, Vol. 1, English translation, Weisbach's Mechanics, by Coxe. Van Nostrand, New York.

thin plates of 1 and 1·41 centimeters, viz. : ·394-inch and ·591-inch in diameter, under heads varying from 10 to 70 feet, are indicated.

TABLE XVII.

Height h, due to velocity, in feet.	Feet 10.	Feet 20.	Feet 30.	Feet 40.	Feet 50.	Feet 60.	Feet 70.
Height of jet projected through circular orfice in a thin plate 0·384 inch=1 centimetre in diameter....................	9·61	18·31	25·98	32·58	38·12	42·66	46·30
Height of jet projected through circular orifice in a thin plate 0 5655 inch=1·41 centimetres in diameter............	9·715	18·69	26·75	33·77	39·72	44·63	48·58

The reduced elevation of 46·30 feet above the plane of the orifice, to which a jet of 1 centimetre is said to reach, when the head of water in the reservoir is 70 feet, is, of itself, very remarkable and cannot well be accounted for solely by the resistance offered by the air, and the so-called resistance encountered during the passage through the orifice, while admitting, in accordance with the theory based on Toricelli's theorem, that the vein should rise to the level of the water surface in the reservoir.

Let us suppose the coefficient of resistance ς produced by the passage of the vein through the atmosphere to be equal to that of a plane surface moving through air, the area of which is equal to that of the cross-section of the vein at every point of its path, viz., to 1·25, according to Du Buat and Thibault.* As air, at the ordinary atmosphere pressure, weighs about $\frac{1}{800}$ of water or, say twice as much, viz., $\frac{1}{400}$, to make ample allowance for any air that may be carried along with the vein, the diminution of the effective pressure of the water due to the passage of the jet through the atmosphere is thus roughly, for 70 feet head of water, $70 \times 1{\cdot}25 \times \frac{1}{400} = 0{\cdot}2187$ feet. Hence, the jet should rise to 69·78 feet, or thereabouts, instead of only 46·30 feet, if the atmosphere was the only resistance to be overcome.

Another proof of the fallacy of attributing to the resistance of the air, the greater part of the difference between the head due to the velocity actually generated in a fluid projected through a simple orifice, and the total fall from the surface in the supplying reservoir to the centre of this orifice, is obtained by comparing Michelotti's experiments on horizontal jets, with those of Dr. Weisbach, on vertical jets.

According to Michelotti, jets issuing from an orifice in a thin vertical plate, 0·889 foot=9 668 inches in diameter, under heads varying from 7·51 to 23·59 feet, and passing therefore, roughly, from 33 to 23 feet through the air, are said to be projected horizontally in each case to a distance equal, within 1 per cent., or less to the corresponding ordinate of the parabola which would be described by the jet if its horizontal velocity near the plane of the orifice was equal to that due to the head.

Weisbach's experiments on vertical jets formed in an orifice 1·41 centimetres or, say $\frac{5}{8}$-inch diameter, under heads of 30 to 40 feet and passing 26·75 to 33·77 feet through the air, go to show that the heights reached by the jets will fall short, in each case of the height of the water surface in the reservoir above the orifice, from 11 to 16 per cent.

I am aware, of course, that a vein formed through an orifice of 9·688 inches is very much larger than one through on opening whose diameter is only $\frac{5}{8}$-inch or so; but I cannot see how even this large difference of area could render the proportional resistance of air [illegible] ten to fifteen times greater in one case than in the other.

As for "the resistance during the passage through the orifice" to which frequent allusion is made in works of hydraulics, I confess that I fail to conceive how it can be possible for any round hole pierced through a plate so thin that it may be

*See English translation Weisbach's Mechanics, page 1031.

considered to be devoid of thickness, to offer resistance to bodies passing through it when ejected from a vessel, no matter what may be the rate of the motion imparted to them.

But then it may be, of course, that after having assumed that theoretically the liquid particles must of necessity acquire, at the short distance of, say one radius of the orifice, in front of the said orifice, a velocity equal to that due to the fall from the surface of the water to the centre of this orifice, the authors, when saying "during the passage through the orifice," mean to refer to the time occupied by the water in passing from within the reservoir to the section of maximum contraction and velocity, or to some other point.*

*If some such broader meaning is attributed to the expression "during the passage through the orifice," I must acknowledge that it is well suited for smoothing over the difficulty of reconciling the shortcomings of a defective theory with the irrefutable arguments supplied by properly substantiated experimental truths.

Although I have not found it practicable, up to the present time, in directly employing equation (6_a) for the computation of the height h, to which a jet will rise vertically in the air under a given head, I am satisfied that the great differences between the height to which the jets experimented on by Dr. Weisbach rose and the corresponding elevations of the water surface in the reservoir of supply. must be attributed chiefly to the decrease ~~in~~ the ~~coefficient of the~~ velocity head of efflux, $\left(\substack{\text{vel.}\\\text{head}\\\text{orif.}}\right)$, and the simultaneous increase of $i_{\binom{v}{a}}$ due to the mutual interference of particles, when we pass from small to great velocities and from large to small orifices.

The following attempts at applications of equation (6_a) for the purposes of discovering what values have to be assumed for $i_{\binom{v}{a}}$ for arriving at the heights to which Dr. Weisbach's jets projected through an orifice 0·394 inch diameter, rose under heads of 10 and 70 feet respectively, ~~as also the diameters at the upper extremities of these jets~~, go to show that this formula does not lead to or absurd results.

In the case of a jet formed in an orifice of 0·394 inch diameter, under a head of 10 feet, we may, judging by what we have seen, put $i_{\binom{v}{a}}$ $s = r = \frac{\cdot 394}{2} = 0{\cdot}197$ inch $=$ 0·016 foot, also $\left(\substack{\text{coeff}\\\text{vel}\\\text{head}\\\text{orif}}\right) = 0{\cdot}61^2 = 0{\cdot}372$, without much risk of material error. These numbers being substituted for the symbols in equation (6_a), it is found that in order that x may be 9·61 feet, $i_{\binom{v}{a}}$ must be equal to 0·40 nearly.

When the diameter of the circular orifice is 0·394 inch and the head 70 feet, we can put $i_{\binom{v}{a}}$ s the distance from the plane of the orifice to the plane of rest, equal to $0{\cdot}6r$, or say 0·01 feet; also $\left(\substack{\text{coeff}\\\text{vel}\\\text{head}\\\text{orif}}\right) = 0{\cdot}58^2 = 0{\cdot}3364$. Upon the respective symbols being replaced by the corresponding numbers in equation (6_a) we find that in order that x may be 46·30 feet $i_{\binom{v}{a}}$ must be equal to about 0·50.

The mean values of $i_{\binom{v}{a}}$ thus established roughly, viz., 0·40 and 0·50 are not absurd or unreasonably low or high, when compared with the mean value of this quantity (0·4096) in the horizontal vein projected through an orifice 0 53 inch in diameter under a head of 14 inches which was photographed, and with that (0·44) in the vertically descending vein projected through an orifice 0·4 inch in diameter under a head of 2·99 inches, which I measured directly with the pointed screws mounted on a diaphragm &c., &c., as explained.

It is not improbable that vertical jets produced under great pressures through orifices in thin plates rise in the air to elevations much below those which jets issuing under the same pressure from properly proportioned conoïdal-mouth pieces would attain, on account of the interference with free efflux from the reservoir, arising in each case from the fact of the body of liquid intervening within the vessel between the surface of the conoïdal form that would be assumed by a theoretically perfect vein and the inner surface of the orifice plate being drawn up in the jet spurting through the orifice in the thin plate.

As stated before, for practical purposes, the coefficient of discharge proper to an orifice in a thin plate may be considered to be invariable, whatever direction the stream flowing through this orifice may follow; in point of fact, however, the discharge through such an orifice must be less, especially under small heads, when the water flows vertically upwards than when it follows a horizontal direction at its exit from the reservoir, notwithstanding the gradual spreading of the fillets, which takes place necessarily in such case from the plane of the orifice to the upper end of the vein, being the result of the action of gravity in a direction contrary to that of the motion of the liquid.

Lorgna says, in article L of his "Phisico Mathematical Theory," etc.:—"It is observed that the quantity of water supplied by a vertical jet, in a fixed time, through a given orifice, and under a permanent head, is much smaller than that which would issue from a reservoir in the same time, through the same orifice, pierced in a thin plate in the side of this reservoir, under the same head." (See the comparison of these discharges in the tables given by M. Bossut, in his Hydrodynamics, Part II., Chap. IV.

DISCHARGE THROUGH CYLINDRICAL AJUTAGES OR TUBES.

Poleni has made known the singular effects of cylindrical tubes two centuries ago; and the determination of the cause has been a serious study with scientists ever since.

If we prevent or destroy, artifically, the inflection of the fillets of a naturally contracted horizontal vein projected through a vertical orifice in a thin plate O R (Fig. 13), by causing this vein to flow through a cylindrical tube O R S T, added to the reservoir at the orifice, so as to completely fill the tube, the velocity acquired by the liquid, and consequently the discharge, in a given time, under a constant head, can be arrived at—if the effects of gravity outside the reservoir are abstracted or neglected—in the manner hereafter described, by supposing the natural fluid fillets to be spread over the full cross-section of the tube in a uniform and continuous manner, by virtue of their attraction towards its sides, in every part of the cylindrical space from O to S—which is not strictly the case in reality, however, as we will see presently. In these conditions the ever-varying ratio between the two velocities which are due to the forces $f i_o$ and $f i_c$ in the natually contracted vein, is continually transformed into the constant ratio of unity or 1, through the intervention of the capillary attraction of the metal, wood, glass, etc., of which the tubular envelope is made, the acceleration due to the force $f i_o$ being increased, and that due to the force $f i_o$ simultaneously diminished in a corresponding manner.

Fig. 13

Thus, if the acceleration due to the force $f i_o$ is continually increased, along the trajectory of the naturally contracted vein abstracted from gravity, in the ratio of 1 to j, the total amounts of momentum due to two sensibly constant mean forces $f i_c$ and $f i_o$ being necessarily the same under all circumstances, at the end of equal times, independently of any transformation whatever, which the constituent factors of mass and velocity may be subjected to within the tube, by virtue of the attraction of its walls—while the momentum is being generated—it follows, that the relation:

$$\begin{pmatrix}\text{vel}\\\text{ratio}\\\text{nat.}\\\text{vein}\end{pmatrix} = \frac{\sqrt{i_{\left(\frac{v.}{a.}\right)} s_0 + x}}{\sqrt{i_{\left(\frac{v.}{a.}\right)} s_0 + i_{\left(\frac{v.}{a.}\right)} x}}$$

which holds good for any point P of the naturally contracted vein, situated at a distance x from the orifice, measured in a direction parallel to the longitudinal axis, will become transformed or converted into the relation :

See page 37

$$\binom{\text{vel ratio}}{\text{cylin stream}} = \frac{\sqrt{i_{\binom{v.}{a.}} s_o + x + (\sqrt{i_{\binom{v.}{a.}} s_o + i_{\binom{v.}{a.}} x} - \sqrt{i_{\binom{v.}{a.}} s_o + i_{\binom{v.}{a.}} jx)}}}{\sqrt{(i_{\binom{v.}{a.}} s_o + i_{\binom{v.}{a.}} jx}} = 1 \tag{11}$$

whence we deduce the equation:

$$\sqrt{i_{\binom{v.}{a.}} s_o + i_{\binom{v.}{a.}} jx} = \sqrt{i_{\binom{v.}{a.}} s_o + x} + \sqrt{i_{\binom{v.}{a.}} s_o + i_{\binom{v.}{a.}} x} - \sqrt{i_{\binom{v.}{a.}} s_o + i_{\binom{v.}{a.}} jx}$$

and the value of j in terms of s_o, i and x, viz.:

$$j = \frac{-s_o}{2x} + \frac{1}{4 i_{\binom{v.}{a.}}} + \tfrac{1}{4} + \tfrac{1}{2}\sqrt{\frac{s_o^{\,2}}{x^2} + \frac{s_o}{x} + \frac{s_o}{i_{\binom{v.}{a.}} x} + \frac{1}{i_{\binom{v.}{a.}}}} \tag{12}$$

Now, if we leave the acceleration due to the force $f i_c$ entirely out of consideration for the present, it will be seen that the total velocity which is due to the force $f i_o$ in the natural contracted vein projected through an orifice in a thin plate, at the instant when the water reaches the point P, bears to the total velocity due to the force $j f i_o$, as increased by the lateral capillary attraction at the inner surface of the cylindrical envelope, the ratio of $\sqrt{i_{\binom{v.}{a.}} s_o + i_{\binom{v.}{a.}} x}$ to $\sqrt{i_{\binom{v.}{a.}} s_o + i_{\binom{v.}{a.}} jx}$. Therefore, also, the quantity of liquid particles, considered for the moment as being independent solid bodies or molecules, that would pass in the unit of time at the point P, on the axis of the contracted stream, by virtue of the velocity generated by the force $f i_o$ from o while a space equal to $i_{\binom{v.}{a.}} s_o + i_{\binom{v.}{a.}} x$ is described and corresponding there, fore to $\sqrt{i_{\binom{v.}{a.}} s_o + i_{\binom{v.}{a.}} x}$ must bear to the volume of molecules that pass in the same time at the same point P, on the axis of the stream rendered artificially cylindrical by means of a tube, by virtue of a velocity corresponding to $\sqrt{i_{\binom{v.}{a.}} s + i_{\binom{v.}{a.}} jx}$, the same ratio of $\sqrt{i_{\binom{v.}{a.}} s_o + i_{\binom{v.}{a.}} x}$ to $\sqrt{i_{\binom{a.}{a.}} s_o + i_{\binom{v.}{a.}} jx}$.

Consequently, abstracting all variations in the resistances of viscosity, friction, etc. due to the altered conditions of the disturbed and partly broken fluid filaments flowing within the tube, as compared to those of the transparent crystal like naturally contracted vein, the mean velocity in the plane of the orifice in a thin plate is to that in the cross section of a cylindrical tube x inches long, or which is the same thing, the discharge through the circular orifice is to the discharge through the cylinder, as $\sqrt{i_{\binom{v}{a}} s_o + i_{\binom{v}{a}} x}$ is to $\sqrt{i_{\binom{v}{a}} s_o + i_{\binom{v}{a}} jx}$.

Hence, in a cylindrical tube l inches long running full, the mean velocity of the stream corresponding to any cross section of the tube is:

$$V_{cyl} = \frac{\sqrt{2g \binom{\text{coeff. vel}}{\text{head orif}} H \left(i_{\binom{v}{a}} s_o + i_{\binom{v}{a}} jl\right)}}{\sqrt{i_{\binom{v}{a}} s_o + i_{\binom{v}{a}} l}}, \text{ or}$$

replacing j by its value in terms of $x = l$ as per equation (12), we have:

$$V_{cyl} = \frac{\sqrt{2g \binom{\text{coeff vel}}{\text{head orif}} H \left\{ s_o + l\left(-\frac{s_o}{2l} + \frac{1}{4 i_{\binom{v}{a}}} + \tfrac{1}{4} + \tfrac{1}{2}\sqrt{\frac{s_o^2}{l^2} + \frac{s_o}{l} + \frac{s_o}{i_{\binom{v}{a}} l} + \frac{1}{i_{\binom{v}{a}}}}\right)\right\}}}{\sqrt{s_o + l}} \tag{13}$$

where $\binom{\text{coeff. vel.}}{\text{head orif.}}$ represents the coefficient (*see* column 5 table XIII), by which the theoretical head H must be multiplied to obtain the head due to the actual velocity in an orifice in a thin plate having the same diameter as the cylindrical tube.

Wherefore, we have finally for the coefficient of discharge $c_{\binom{\text{disc:}}{\text{cylin:}}}$ of the cylin-

drical tube as compared to a coefficient of discharge equal to unity, or 1 for the simple orifice in a thin plate :

$$c_{\binom{\text{disc.}}{\text{cylin.}}} = \frac{v_{\text{cyl.}}}{v_{\binom{\text{simple}}{\text{orifice}}}} = \frac{\sqrt{s_0 + l\left(-\frac{s_0}{2l} + \frac{1}{4i_{\binom{v}{a}}} + \frac{1}{4} + \frac{1}{2}\sqrt{\frac{s_0^2}{l^2} + \frac{s_0}{l} + \frac{s_0}{i_{\binom{v}{a}}\,l} + \frac{1}{i_{\binom{v}{a}}}}\right)}}{\sqrt{s_0} + l} \tag{14}$$

EXAMPLE 1.

By using a cylindrical tube, such as that represented in (Fig. 16), 18 old french lines = 1·5985 inches in diameter, but only 54 lines = 4·7955 inches long, Venturi obtained under a constant head of 32·5 french inches = 34·6476 english inches, a discharge from the reservoir, bearing to that passing under the same head, through a circular orifice in a thin plate having the same diameter as the tube, the ratio of 41 to 31*. The delivery of 4 cubic feet took the same time, viz , 31 seconds, when the tube was 57, instead of only 54 lines. (See page 136, exp. 6, same work.)

In the case of the vein projected under a head of some 14 inches through an orifice in a thin plate 0 53-inch diameter, which was photographed s_0, was found to be approximately equal to 0·57 r, r being the radius of the orifice. If we assume, therefore, s_0 to vary nearly inversely as the square root of the velocity, we can here put $s_0 = \cdot 57\, r\left(\frac{\sqrt[4]{14}}{\sqrt[4]{34\cdot 64}}\right) =$ say $\cdot 45\ r =$ say 4·00 lines. Again, we may allow, in the absence of any more precise data, that for a diameter of 1·5985 inches, and a head of 34 64 inches, $i_{\binom{v}{a}}$ has nearly the same value as for an orifice of 0·4 inch diameter, and a head equal to $34\cdot 64 \times \left(\frac{\cdot 40}{1\cdot 5985}\right) =$ say 8·7 inches, when, according to experiments Nos. 15, 16, 17, 18 and 19, of Table V, we may put approximately $i_{\binom{v}{a}} = c_c^4 = 0\cdot 42$ on an average, along the portion of natural vein 54 lines or 4.7955 inches long, which corresponds, as regards position with reference to the orifice and reservoir, to the cylindrical tube.

Substituting these numbers for the respective symbols in the last equation (14), we find the computed velocity ratio $c_{\binom{\text{disc.}}{\text{cylin}}} = \frac{v_{\text{cylin.}}}{v_{\text{simple orifice}}}$ to be 1·26, as against $\frac{41}{31} = 1\cdot 32$ obtained by direct experiment, indicating a deficiency of about 5 per cent. in the computed velocity.

While a small part of this difference may be the result of the disengagement of the fluid particles produced by the attraction of the sides of the tube, and of the transverse action of gravity during the passage of each sheet of water from the reservoir end O R (Fig. 13), to the other extremity S T, of the tube, the greater portion of it is, in all probability, due to the fact that the filaments of the naturally contracted vein are not dispersed in a uniform and continuous manner over the entire cross-section of the cylinder, as was assumed, at least for a length of one diameter or so beyond the face O R of the reservoir. The actual conditions of the flow through the simple tube are apparently intermediate between the theoretical conditions upon which the above computation is based and those of a stream flowing through a divergent tube of the form $o\, r$ S T (Figs. 14) added to a mouth-piece $o\, r$ O R having the shape of the naturally contracted vein. 15

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* See Experimental Enquiries, concerning the principles of the lateral communication of motion in fluids, applied to the explanation of various hydraulic phenomena, by citizen J. B. Venturi, translated from the French by W. Nicholson ; second edition, included in Tracts on Hydraulics, edited by Thos. Tredgold, page 134, London, printed for Josiah Taylor, 1826.

EXAMPLE 2.

Buff* found that with a short cylindrical tube $\frac{3}{10}$ inch in diameter and $\frac{5}{10}$ inch long the coefficient of discharge was 0·861 under a head of $2\frac{1}{2}$ inches. As the coefficient of discharge into air through a simple orifice of the same diameter as the tube and under the same head, may be taken at 0·65 nearly, the ratio of the discharging capability of the tube to that of the simple orifice in a thin plate is $0{\cdot}861 \div {\cdot}650 = 1{\cdot}3246$.

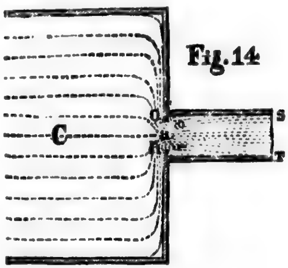
We may in this case put approximately $s_0 = {\cdot}9\,r = {\cdot}135$ inch, and $i = {\cdot}41$, whence, substituting these values for the symbols, in the above formula and 0·50 inch for l, we find this ratio to be equal to 1·23 nearly.

The difference between the observed and the computed coefficient of velocity is therefore ·0946, indicating a deficiency in the latter coefficient of some 8 per cent., due to the causes just described.

The increased discrepancy of 8 per cent, as compared to that of 5 per cent. in example 1, is, I presume, due here to the greater transverse effect of gravity in the cylindrical vein—during its passage from the reservoir to the outer end of the tube, with the comparatively small velocity generated by a head of $2\frac{1}{2}$ inches.

I have taken the liberty to introduce here in extenso a chapter from Hydraulic Tables, Coefficients and Formulæ, by John Neville, Esq., Civil Engineer, M.R.I.A., &c., &c., on the conditions of flow, &c., in short cylindrical tubes, with and without entrance contracted by a diaphragm, wherein a method is suggested for calculating the discharge from such tubes. This course was followed with a view to convenience for reference, &c., in perusing some remarks which I have ventured to offer respecting some of the statements, etc., contained in the said chapter.

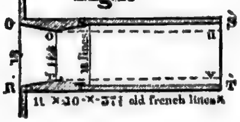
At pages 160 to 164 of Mr. Neville's valuable work, 3rd edition, dated London, 1875, we find the following:—



The contracted vein $o\,r$ is about 0·8 times the diameter O R; but it is found, notwithstanding, that water in passing through a short tube of not less than $1\frac{1}{2}$ diameter in length, fills the whole of the discharging orifice S T. This is partly effected by the outflowing column of water carrying forward and exhausting the air between it and the tube, and by the external air then pressing on the column, so as to enlarge its diameter and fill the whole tube. When once the water approches closely to the tube, or is caused to approach, it is attracted and adheres with some force to it. The water between the

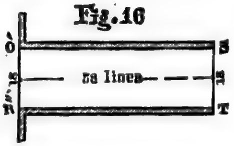
* Annalen der Phisik and Chemie Von Poggendorf, 1839, Band 46, page 243, or Neville's Hydraulic Tables, coefficients and formulæ, page 148. Third Edition. London, 1875.

Fig. 15



tube and the *venâ contractâ* is, however, rather in a state of eddy than of forward motion, as appears from the experiments of *Venturi with the tube shown Fig. 15, giving the same discharge as the simple cylindrical tube (Fig. 16.)

Fig. 16



where $O'R = O''R$, $O'S = O''S$, $S'T = S''T$. If the entrance be contracted by a diaphragm, as at O R, Fig. 14, the water will also generally fill the tube, if it be only sufficiently long. Short cylindrical tubes do not fill when the discharge takes place in an exhausted receiver, but even diverging tubes will be filled under atmospheric pressure when the angle of divergence *o*, does not exceed 7 or 8 degrees, and the length be not very great nor very short.

Fig. 17

When a tube is fitted to the bottom or side of a vessel it is found that the discharge is that due to the head measured from the surface of the water to the lower or discharging extremity of the tube. It must, however, be sufficiently long, and not too long, in order to get filled throughout. Guilglielmini first referred this effect to atmospheric pressure, but the first simple explanation is that given by Dr. Mathew Young, in the Transactions of the Royal Irish Academy, Vol. VII., page 56. Venturi, also, in his fourth proposition, gives a demonstration.

The values of the coefficients for short cylindrical tubes, which are given (page 156), have been derived from experiments. Coefficients which agree pretty closely with them, and which are derived from the coefficients of discharge through an orifice in a thin plate, may, however, be calculated as follows: Let C be the area of the

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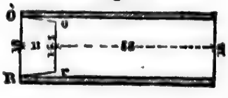
* Venturi found (1) that through an orifice O R pierced in a thin plate in the side ~~the~~ of a reservoir, whose diameter was 18 French lines (old system of measures) = 1.5985 English inches — 4 French cubic feet = 4.8384 English cubic feet of water are discharged in 41 seconds under a head of 32.5 French inches = 34.6476 English inches. (2.) He fitted to this orifice the conical mouth piece O R, *o r*, of the proportions shown in Fig. 15, and having nearly the form of the natural contracted vein, when under the same head the same quantity of water was discharged in 42 seconds. (3.) By introducing the mouth-piece O *o*, *r* R (Fig. 15) alone into the cylindrical tube Fig. 15, as shown in Fig. 18, the same volume of water was discharged in 32.5 seconds. (4.) To the mouth-piece O *o*, *r* R, he added the tube *o* S S T T*ro* (Fig. 15), and the duration of the flow, all other things being equal was only 31 seconds. (5.) He replaced the compound tube O *o* S S T T *r* R O by the simple cylindrical tube Fig. 16, having the same diameter and length, and the efflux of 4.8384 feet took place again in 31 seconds. (6.) Lastly, when he had amended the portion *o* S T *r o* (probably by rounding the angles at *o* and S) the time required for discharging the constant quantity of 4.8384 cubic feet was reduced to 30 seconds, under the same head of 24.6476 inches.

Fig. 18



approaching section (Fig 14), A the area of the discharging short tube, and a the area of the orifice O R, which admits the water from the vessel into the tube: also put, as before, h for the head measured from the surface of the water to the centre of the tube and diaphragm O R; v for the actual velocity of discharge at S T; v_a for the velocity of approach in the section C towards the diaphragm O R; and c_c for the coefficient of contraction in passing from O R to $o\,r$; then $C \times v_a = A \times v$, the contracted section $o\,r = c_c \times a$, and consequently the velocity at the contracted section is equal to $\frac{A\,v}{a\,c_c} = \frac{C\,v_a}{a\,c_c}$. Now a theoretical head equal to $\frac{v^2 - v_a^2}{2g} = \frac{v^2\left(1 - \frac{A^2}{C^2}\right)}{2g}$ is necessary to change the velocity v_a into v by the action of gravity; but as the water at the contracted section $o\,r$, moving with a velocity $\frac{A\,v}{a\,c_c}$, strikes against the water between it and T S, moving, from the nature of the case, with a slower velocity,* a certain loss of effect takes place from impact. If this be supposed sudden, then writers on mechanics have shown *that a total loss of head, equal to that due to difference of the velocities,* $\frac{A\,v}{a\,c} - v$, *before and after the impact must take place.*

This loss of head is therefore equal to $\frac{\left(\frac{A}{a\,c_c} - 1\right)^2 v^2}{2g}$, whence the whole head.

$$(60.) \qquad h = \frac{\left(1 - \frac{A^2}{C^2}\right) v^2 + \left(\frac{A}{a\,c_c} - 1\right)^2 v^2}{2g}$$

from which the velocity from a short tube is found to be

$$(61) \qquad v = \sqrt{2gh} \left\{ \frac{1}{1 - \frac{A^2}{C^2} + \left(\frac{A}{ac_c} - 1\right)^2} \right\}^{\frac{1}{2}}.$$

Now as $\sqrt{2gh}$ would be the velocity of discharge were there no resistances or loss sustained it is evident that $\left\{ \frac{1}{1 - \frac{A^2}{C^2} + \left(\frac{A}{ac_c} - 1\right)^2} \right\}^{\frac{1}{2}}$ becomes as it were a coefficient velocity. When the diameter of the diaphragm O R, becomes equal to the diameter S T of the tube, $A = a$, and as the coefficient of velocity becomes equal to the coefficient of discharge when there is no contraction, in such case this coefficient which we call *cof*, is expressed by the formula

$$(62) \qquad \text{cof.} = \left\{ \frac{1}{1 - \frac{A^2}{C^2} + \left(\frac{1}{c_c} - 1\right)^2} \right\}^{\frac{1}{2}}$$

When the diaphragm is placed in a tube of uniform bore, then C=A and

$$(62\tfrac{1}{2}) \qquad \text{cof.} = \frac{1}{\frac{A}{a\,c_c} - 1} = \frac{c_c}{\frac{A}{a} - c_c}.$$

and the loss of head in passing the diaphragm becomes:

$$(62\tfrac{3}{4}) \qquad n = \left(\frac{A}{ac_c} - 1\right)^2 \times \frac{v^2}{2g.}$$

* *Vide* Sir Robert Kane's translation of Rühlman's book on Horizontal Water Wheels, p. 40.

It is evident from the equations that $\frac{A}{a}$ and c_c depend mutually on each other, and that they cannot be assumed arbitrarily.

When the approaching section C is very large compared with the area A

$$(63) \qquad \text{cof.} = \left\{ \frac{1}{1 + \left(\frac{1}{c_c} - 1 \right)^2} \right\}^{\frac{1}{2}}$$

If $c_c = 0{\cdot}64$, the last equation gives *cof.* $= {\cdot}872$; if $c_c = {\cdot}601$ *cof.* $= {\cdot}833$; if $c_c = {\cdot}617$ *cof.* $= {\cdot}847$; and if $c_c = {\cdot}621$ *cof.* $= {\cdot}856$. These results are in excess of those derived from experiments with cylindrical short tubes, perfectly square at the ends and of uniform bore. As some loss, however, takes place in the eddy between *or*, Fig. 14, and the tube, and from the friction at the sides, not taken into account in the above calculation, they will account for the difference of not more than from 4 to 5 per cent. between the calculation and experiment. If c_c be assumed for calculation equal ·590, then *cof.* equals ·821; and as this result agrees very closely with the experimental one, c_c should be taken of this value in using the foregoing formulae, from (60) to (63 for practical purposes. The thickness of the diaphgram itself and the relation of that thickness to the diameter, as well as the form of the orifice *a*, are necessary elements in the consideration of this question."

REMARKS.

Considering that the natural contraction of the liquid vein projected through a simple orifice, is destroyed gradually in a cylindrical tube, from a point between the orifice O R in the reservoir, and the section of maximum contraction *or* (Fig. 14) up to the point to which the tube must extend to furnish a full stream, the water in this contracted section *or*, cannot, it seems to me, be looked upon as striking *suddenly* against the body of water between it and the end section T S, hence the consequent reduction in the total head cannot be exactly the amount of pressure corresponding to the difference between the total theoretical velocity due to the full head and the actual velocity of the stream at its exit from the tube.

Streams passing from short cylindrical tubes into the open atmosphere invariably carry a certain quantity of air along with them, and in order that air may be able to mix with the water, it is necessary that the absolute pressure of the vein at the mouth of the tube should be different from that of the atmosphere. From this circumstance it must not be inferred, however, that the presence of atmospheric air, or some other gaseous fluid in the tubes, is essential, in order that the filling of the same may take place, with the resulting increased discharges in comparison to those afforded by simple orifices of equal diameters and under the same respective hydrostatic pressures; the air or any other gas that may be in the tubes, no doubt, assists in causing these to fill with water, but that is all.

The statement that "cylindrical tubes do not fill when the discharge takes place "in an exhausted air receiver," is apparently incorrect, for Mr. Hachette says he is certain of having produced the phenomena of additional tubes under such a receiver, in vacuum.* The same experimenter also managed to obtain a clear, contracted vein within a cylindrical tube 0,1332 ft. diameter, and 0,3117 ft. long, which was perforated near its middle and quite around its perimeter with a dozen small holes; but this operation, it is stated, had to be performed with great caution, as a slight agitation was then sufficient to produce contact, causing a flow with full tube to take place.

I have seen no detailed description of the experiments made by Mr. Hachette. It would be interesting to know what the pressure was in a cylindrical tube running full, say at a distance of half a diameter or so from the orifice in the reservoir, when the pressure in the receiver of the air pump was down to near 0. According to the theory

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*See Spon's Dictionary of Engineering, page 1,901.

of Daniel Bernouilli, that the pressure which a fluid exerts against the sides of a tube in which it moves, is equal to the head, minus the height due to the velocity of the motion, the absolute pressure in Mr. Hachette's tube, near the spot pointed out, must, under such circumstances, have been less than 0, provided that the head of water used in making the experiment exceeded, say $1\frac{1}{2}$ times, the small tension which could not be eliminated from the receiver,—that is to say, the exhausting power of the stream must have been greater than the minimum power of aspiration capable of producing or forming what is termed to be a vacuum, viz., a space devoid of ponderable matter of any kind, air included. Now, the internal condition of such a stream of water must be different, at least as regards absolute tension, from that of the space freed of all matter, which we call a vacuum; the question therefore presents itself: In what manner does an increase in the power of exhaustion, of a liquid vein touching the sides of a cylindrical tube, effect the conditions of molecular equilibrium of the substance, if any, that fills a space enclosed by a vessel placed in communication with the tube, after all ponderable matter, air included, is exhausted therefrom.

However this may be, I am inclined to believe that the increased discharge afforded by cylindrical and divergent tubes, is entirely due to the spreading action brought about by the adhesive or attractive properties of their sides or envelopes, by virtue of which the relations between the inertia and attraction or cohesion of the particles of ponderable matter moved, are continually modified in the tube during the gradual enlargement of the sectional area embraced by the stream, the tendency being to create an absolute vacuum—and that the pressure of the atmosphere is not essential to the successful production of this state.

Venturi was mistaken in attributing the increased discharge to an excess in the pressure of the atmosphere on the fluid surface of the reservoir, viz.: an excess proceeding from a vacuum tending to arise in the part of the tube where the greatest contraction took place; the partial vacuum produced in every case of efflux through such a tube is only the effect of the real cause of such increased discharge.

The fact of the compound tube, fig. 15, discharging, under a constant head, an equal volume of liquid in the same time as the simple cylindrical tube, fig. 16, coupled with the result, showing: that with the compounded tube, a little less time was required to supply the same quantity of water in the conditions—all of which tubes have the same diameter at the ends, and also the same length along the axis—does not strike me as being conclusive evidence that the space between the envelope of the first named tube (fig. 15) and the natural contracted vein, or *venâ contractâ*, is occupied by eddy water *causing, on the whole, a sensible loss of velocity* in the stream flowing through this simple tube.

It appears to me that a smooth cylindrical channel, by the gradual attraction of the liquid fillets towards its sides, tends to produce an effect equivalent to that which would result from the application to the orifice ŌR Fig. 16 of a compound tube of a total length ŌS not exceeding that of the cylinder, composed of a conoidal mouthpiece with divergent extension of maximum discharging power, and at the same time the tube lessens the chances of mutual interference of particles; facilitating the passage of the excessively convergent conoidal vein ejected through an orifice with sharp edges, quite as much, if not more so, as the eddy water lodging in the said tube may obstruct it. I believe, that on the whole, instead of being slower in the cylindrical tube, the motion of the liquid fillets passing within the conoidal space swept out by the horizontal contracted vein is quite as rapid, independently of any additional acceleration due directly to the spreading out of the stream towards the sides of the tube, as that of the corresponding fillets of the naturally contracted vein. Instead of being in excess of the values derived from experiments, by from 4 to 6 per cent., the computed values of cof. (by means of equation 63) should, therefore, have proved deficient to about the same extent.

Where Mr. Neville says: " *When a tube is fitted to the bottom or side of a reservoir, it is found that the discharge is that due to the head measured from the surface of the water to the lower or discharging extremity of the tube,*" he must mean, no doubt, a cylindrical tube fitted to a convergent conoidal mouthpiece having the form of the contracted vein, for he refers to Venturi's fourth proposition as a proof of the correctness of this law.

The velocities at the lower ends of such tubes, fitted to conoidal convergent mouth pieces, happen to agree tolerably well with those acquired by solid bodies after these have descended freely through spaces equal in each case, to the head measured from the surface of the water to the discharging extremity of the tube. These coincidences do, however, in my opinion, no more possess the fundamental character which it is sought to attach thereto than that other, if anything, more generally accepted so-called hydraulic law; "the velocity of a fluid at its passage "through an orifice, made in the side or bottom, or top of a reservoir, is the same as "that which a heavy body would acquire in falling freely through a space equal to "that comprised between the level of the fluid surface in the reservoir and the centre "of that orifice"—the acceptance of which experimental indication as a natural law, the celebrated Lorgna has conclusively shown —not to be warranted by the facts and truths elicited by properly directed investigation.*

In their attempts at theoretical demonstrations of the law just enunciated, modern authors, in general, shelve all difficulties apparently without any scruples, by constructing a reservoir to suit themselves, viz., one having its sides joined with the orifice of efflux by easy convergent channels of approach, in order that, they state, the parallelism of the moving sheets or layers of liquid taken perpendicularly to the axis of the steam can be considered to be perfectly realized; it is clear, however, that this is in reality equivalent to dodging past the contracted vein, which, however, unwilling they may be to admit it, remains the stumbling block in their way.

DISCHARGE THROUGH DIVERGENT AJUTAGES OR TUBES.

1. Tubes A B C D A, applied directly to the Wall of the Resevoir, without the Intervention of a Conoïdal Mouth-piece, having the form of the Natural Contracted Vein.

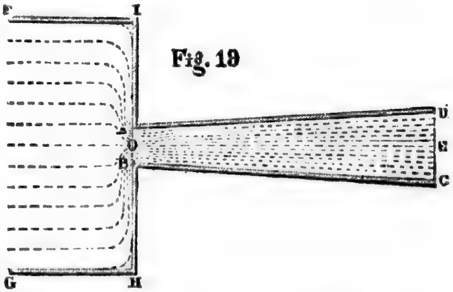


Fig. 19

If, in addition to the abstraction of gravity outside of the reservoir F G H I (Fig. 19), it is assumed, as was done in the case of cylindrical tubes, that the fluid filaments of a naturally contracted vein issuing through an orifice A O B, are dispersed in a uniform and continuous manner over the entire cross-section of the divergent tube A B C D, fitted to the orifice A B, in the reservoir, as shown in Fig. 19 at every point of their path along the axis O E, through this tube — notwithstanding that this hypothesis is even perhaps a little further removed from the true conditions of the efflux through divergent tubes unprovided with conoïdal mouthpieces, than it is from the conditions of the efflux through cylindrical tubes—the coefficient of efflux or discharge proper to a divergent tube such as A B C D, viz., the ratio of this discharge to that afforded in the same time and under the same head, by an orifice A O B, in a thin plate, can be determined as follows:

Here, as in plain cylindrical tubes fitted directly to the wall of a reservoir, the ever-varying ratio between the velocities which are respectively due to the forces fi_o and fi_c in the naturally contracted vein, is continually being transformed through

* See translation of first two chapters of his "Physicomathematical Theory of the motion of liquids issuing from orifices in reservoirs" appended hereto.

the intervention of the capillary attraction of the sides of each tube; the force fi_o being increased, not only in the tubes which are absolutely divergent, but also for tubes whose sides have a less convergence than those of a mouthpiece having the form of the naturally contracted vein—and the force fi_c being simultaneously modified in a contrary sense.

If, therefore, the force fi_o is transformed into $j\ fi_c$, j being any positive number whatsoever, greater than unity—considering that the total amount of momentum which can be developed in an element of mass by any two forces in the unit of time, or during any fixed period of time, must remain constant, so long as there is nothing added to nor subtracted from the sum of the forces—the expression:

$$\frac{\sqrt{i_{\binom{v}{a}} s_o + x}}{\sqrt{i_{\binom{v}{a}} s_o + i_{\binom{v}{a}} x}}$$

which represents, in a general way, the proportional velocity v_p or velocity ratio of the motions due to the two forces fi_c and fi_o at any point of the naturally contracted horizontal veins abstracted from gravity outside of the reservoir, in terms of the abcissa x—becomes converted in the divergent tube into:

$$\frac{\sqrt{i_{\binom{v}{a}} s_o + x} + \sqrt{i_{\binom{v}{a}} s_o + i_{\binom{v}{a}} x} - \sqrt{i_{\binom{v}{a}} s_o + i_{\binom{v}{a}} jx}}{\sqrt{i_{\binom{v}{a}} s_o + i_{\binom{v}{a}} jx}}$$

But here this fraction is not uniformly equal to unity, as was the case for cylindrical tubes.

In all tubes in general, all other things being equal, the proportional (not the actual) velocities, or the velocity ratios v_p of the moving fluid, evidently vary, along the axis, inversely as the areas πy^2 of their circular cross-sections, viz.: as $\frac{1}{y^2}$ so that

$$\frac{v_p}{v'_p} = \frac{\frac{1}{y^2}}{\frac{1}{y'^2}}$$

where v_p is the velocity ratio corresponding to the ordinate y and v'_p that corresponding to the ordinate y'.

But when the length $OE = x$ of the tube A B C D is reduced to o, viz.: when this tube is removed altogether from the reservoir, and the fluid passes through the orifice A O B, we have for the proportional velocity or velocity ratio:

$$v_p = \frac{\sqrt{i_{\binom{v}{a}} s_o + o} + \sqrt{i_{\binom{v}{a}} s_o + i_{\binom{v}{a}} o} - \sqrt{i_{\binom{v}{a}} s_o + i_{\binom{v}{a}} jo}}{\sqrt{i_{\binom{v}{a}} s_o + i_{\binom{v}{a}} jo}} = 1 \qquad (15)$$

Again, in conical tubes such as A B C D, $y^2 = (r + mx)^2$ where r is the radius of the small base and m represents the tangent of the semi-angle of divergence of the sides A D, B C, of the tube. Hence we have the relation:

$$\frac{\dfrac{\sqrt{i_{\binom{v}{a}} s_o + x} + \sqrt{i_{\binom{v}{a}} s_o + i_{\binom{v}{a}} x} - \sqrt{i_{\binom{v}{a}} s_o + i_{\binom{v}{a}} jx}}{\sqrt{i_{\binom{v}{a}} s_o + i_{\binom{v}{a}} jx}}}{1} = \frac{\frac{1}{y^2}}{\frac{1}{r^2}} = \frac{r^2}{y^2} = \frac{r^2}{(r+mx)^2} \qquad (16)$$

whence: $$\sqrt{i_{\binom{v}{a}} s_o + i_{\binom{v}{a}} jx}\left\{1 + \frac{r^2}{(r+mx)^2}\right\} = \sqrt{i_{\binom{v}{a}} s_o + x} + \sqrt{i_{\binom{v}{a}} s_o + i_{\binom{v}{a}} x}$$

and:

$$j = \frac{2i_{\binom{v}{a}} s_o + x + i_{\binom{v}{a}} x + 2\sqrt{i^2_{\binom{v}{a}} s_o^2 + i_{\binom{v}{a}} s_o x + x i^2_{\binom{v}{a}} s_o + i_{\binom{v}{a}} x^2} - i_{\binom{v}{a}} s_o \left(1 + \frac{r^2}{(r+mx)^2}\right)^2}{i_{\binom{v}{a}} x \left\{1 + \frac{r^2}{(r+mx)^2}\right\}^2} \qquad (17)$$

Substituting therefore, in the expression $\dfrac{\sqrt{i_{\binom{v}{a}}s_0 + i_{\binom{v}{a}}jx}}{\sqrt{i_{\binom{v}{a}}s_0 + i_{\binom{v}{a}}x}}$ which represents, as explained in the case of the cylindrical tube, the ratio between the absolute number of liquid molecules passing the plane of the orifice A O B, in a thin plate, during a given time, and that flowing through the corresponding base A O B, of any tube of the length x, during the same time—the value of j just found in terms of x for the symbol, we obtain for the velocity $v\left(\begin{smallmatrix}\text{AOB}\\ \text{div.}\\ \text{conc.}\end{smallmatrix}\right)$ in the small base A O B, of any conical divergent tube A B C D, whose length O E $= l$, applied directly to the side of a reservoir, viz.: without contracted mouthpiece:

$$\left(\begin{smallmatrix}\text{vel}\\ \text{small}\\ \text{base}\\ \text{simple}\\ \text{div}\\ \text{cone}\end{smallmatrix}\right) = \sqrt{\frac{2g\left(\begin{smallmatrix}\text{coeff}\\ \text{vel}\\ \text{head}\\ \text{orif}\\ \text{equal}\\ \text{small}\\ \text{base}\end{smallmatrix}\right)H\left[\dfrac{2i_{\binom{v}{a}}s_0 + l + i_{\binom{v}{a}}l + 2\sqrt{i^2_{\binom{v}{a}}s_0^2 + i_{\binom{v}{a}}s_0 l + li^2_{\binom{v}{a}}s_0 + i_{\binom{v}{a}}l^2}}{\left(1 + \dfrac{r^2}{(r+ml)^2}\right)^2}\right]}{i_{\binom{v}{a}}s_0 + i_{\binom{v}{a}}l}} \qquad (18)$$

Let us now apply this formula to the determination of the velocities at the bases next to the reservoir, of some ofthe conical divergent tubes experimented with, for the purpose of comparing the computed ratio between the velocity in an orifice pierced in a thin plate and that in the small base of the tube, in each case with the corresponding velocity ratio deduced from experimental data.

EXAMPLE.

By fitting immediately to the side of a reservoir, viz.: without intermediate contracted mouthpiece, a divergent tube, whose length O E $= l = 9{\cdot}2124$ inches, was nine times its diameter A B $= 2r = 1{\cdot}0236$ inch at the small end, the flare of its sides A D, B C, being 5°–6′ and the diameter of the large base D C $= 2\,(r = m\,l)$ $= 1{\cdot}8441$ inches, Eytelvein found that with a constant head of 2·3642 feet $= 28{\cdot}37$ inches, the coefficient of discharge for the base A B, was 1·18, the theoretic discharge being 1.

As already done in other cases, we may here assume, without risk of erring materially, that s_0 varies inversely as the square root of the velocity—consequently, as for 14 inches head I found s_0 to be equal to from 0·54 to 0·57 r or so, we have for a head of 28·37 inches, $s_0 = 0{\cdot}57\, r \dfrac{\sqrt[4]{14}}{\sqrt[4]{28{\cdot}37}} = {\cdot}2917 \times \dfrac{1{\cdot}934}{2{\cdot}308} =$ say, 0·25 inch.

Again, judging by the results entered in Tables V and XIII, we may put approximately $i_{\binom{v}{a}} = 0{\cdot}43$ and also $c_{\left(\begin{smallmatrix}\text{vel}\\ \text{head}\\ \text{orif}\\ \text{A O B}\end{smallmatrix}\right)} = 0{\cdot}630^2 = {\cdot}3969$.

Substituting these values for the symbols in equation (18), we obtain 1·21 for the coefficient of discharge at the base A O B, through the tube A B C D, in place of 1·18 found by Eytelvein.

N.B. I applied directly to the horizontal bottom of my circular reservoir, viz., without the intervention of a conoïdal convergent mouth-piece, a conically divergent brass tube 12 inches or nearly 29 diameters long, whose small base was 0·422 inch in diameter and the large base 1·333 inches, the total angle of divergence of the sides being thus 4° 22′, and found the coefficient of discharge under water to be, on an average, 1·12 in the small base of the tube, with an effective head or difference of level between the water surfaces of the supplying and receiving reservoirs of 1·30 inches, as compared to a theoretic discharge of 1 at the same place—and 1·723 as compared to the actual discharge under water through an orifice in a thin plate.

I am not certain, however, that this tube was effective over its whole length—failing which, the lower portion must have proved more of an obstruction to the passage of the water than otherwise.

2° Tubes C D E G applied to the small base D C, of the Conoidal Mouth-piece A B C D, having nearly the form of the Naturally Contracted Vein.

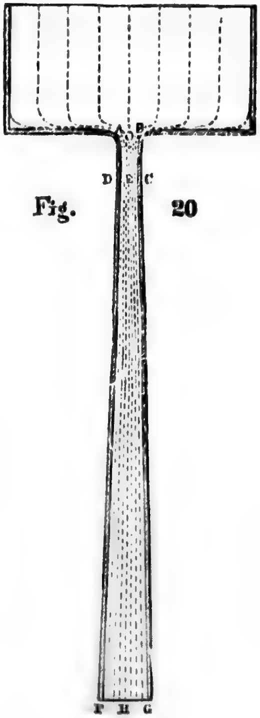


Fig. 20

Here the expression $\dfrac{\sqrt{i_{(\frac{v}{a})} s_0 + x}}{\sqrt{i_{(\frac{v}{a})} s_0 + i_{(\frac{v}{a})} x}}$ which, as shown, denotes correctly, in general terms, the ratio of the velocity or motion proper to any point on the axis of the naturally contracted vein, abstracted from gravity outside the reservoir, to that in the orifice A O B, becomes transformed by virtue of the lateral capillary attraction of the the tubular envelope, only after this natural vein A B C D, has described a portion of the trajectory $x =$ O E, Fig. 20, viz.: into

$$\frac{\sqrt{i_{(\frac{v}{a})} s_0 + x + x'} + \sqrt{i_{(\frac{v}{a})} s_0 + i_{(\frac{v}{a})} x + i'_{(\frac{v}{a})} x} - \sqrt{i_{(\frac{v}{a})} s_0 + i_{(\frac{v}{a})} x + i_{(\frac{v}{a})} j x'}}{\sqrt{i_{(\frac{v}{a})} s_0 + i_{(\frac{v}{a})} x + i_{(\frac{v}{a})} j x'}}$$

where x' represents E H the length of the divergent tube. Now, considering that when the length x' of the divergent tube is reduced to 0, viz., when the tube is removed altogether and the fluid passes only through the mouth-piece A B C D, the proportional velocity or velocity ratio is simply, as shown above, equal to:

$$\frac{\sqrt{i_{(\frac{v}{a})} s_0 + x}}{\sqrt{i_{(\frac{v}{a})} s_0 + i_{(\frac{v}{a})} x}}$$

and, moreover, as the velocity ratios corresponding to any two sections D C and F G, of the compound tube A B G F, must vary inversely as the squares of their diameters or radii, we have the following relation:

$$\frac{\dfrac{\sqrt{i_{\binom{v}{a}}\, s_0 + x + x'} + \sqrt{i_{\binom{v}{a}}\, s_0 + i_{\binom{v}{a}}\, x + i_{\binom{v}{a}}\, x'} - \sqrt{i_{\binom{v}{a}}\, s_0 + i_{\binom{v}{a}}\, x + i_{\binom{v}{a}}\, jx'}}{\sqrt{i_{\binom{v}{a}}\, s_0 + i_{\binom{v}{a}}\, x + i_{\binom{v}{a}}\, jx'}}}{\dfrac{\sqrt{i_{\binom{v}{a}}\, s_0 + x}}{\sqrt{i_{\binom{v}{a}}\, s_0 + i_{\binom{v}{a}}\, x}}}$$

$$= \frac{\mathrm{D\,E}^2}{\mathrm{F\,H}^2} = \frac{r'^2}{(r' + mx')^2} \qquad (19)$$

where r' represents D E, and m the tangent of half the angle included between the sides D F and C G, whence we deduce:

$$j = \frac{\left\{\sqrt{i_{\binom{v}{a}}\, s_0 + x + x'} + \sqrt{i_{\binom{v}{a}}\, s_0 + i_{\binom{v}{a}}\, x + i_{\binom{v}{a}}\, x'}\right\}\left\{s_0 + x\right\}}{x'\left\{\sqrt{i_{\binom{v}{a}}\, s_0 + i_{\binom{v}{a}}\, x} + \left(\sqrt{i_{\binom{v}{a}}\, s_0 + x}\right)\left(\dfrac{r'^2}{(r' + mx')^2}\right)\right\}^2} - \frac{s_0 + x}{x'} \qquad (20)$$

If now we substitute this value of j for this symbol in the expression:

$$\frac{\sqrt{i_{\binom{v}{a}}\, s_0 + i_{\binom{v}{a}}\, x + i_{\binom{v}{a}}\, jx'}}{\sqrt{i_{\binom{v}{a}}\, s_0 + i_{\binom{v}{a}}\, x + i_{\binom{v}{a}}\, x'}}$$

which, as previously explained, represents the ratio which the absolute number of fluid particles, considered as solid molecules, that pass in the unit of time through the orifice in a thin plate A O B, as well as through the section of maximum contraction D E C, bears to the number of particles that flow, under the same conditions and during the same time, through the corresponding bases A O B and D E C of the compound tube A B C G F D A, we obtain for the velocity in the small base D E C of this tube:

$$\left(\begin{smallmatrix}\text{velo.}\\ \text{small}\\ \text{base}\\ \text{div.}\\ \text{tube}\\ \text{with}\\ \text{mouth}\\ \text{piece.}\end{smallmatrix}\right) = \frac{\sqrt{2gH\left\{\dfrac{\left\{\sqrt{i_{\binom{v}{a}}\, s_0 + x + x'} + \sqrt{i_{\binom{v}{a}}\, s_0 + i_{\binom{v}{a}}\, x + i_{\binom{v}{a}}\, x'}\right\}^2\left\{s_0 + x\right\}}{\left\{\sqrt{i_{\binom{v}{a}}\, s_0 + i_{\binom{v}{a}}\, x} + \left(\sqrt{i_{\binom{v}{a}}\, s_0 + x}\right)\left(\dfrac{r'^2}{(r' + mx')^2}\right)\right\}^2}\right\}}}{\sqrt{s_0 + x + x'}}$$

$$\times \left(\begin{smallmatrix}\text{coeff.}\\ \text{velocity}\\ \text{natural}\\ \text{contracted}\\ \text{vein at}\\ \text{D E}\end{smallmatrix}\right) \times \left(\begin{smallmatrix}\text{coeff.}\\ \text{velocity}\\ \text{orifice}\\ \text{D E C}\\ \text{mouth-}\\ \text{piece}\end{smallmatrix}\right) \qquad (21)$$

H standing for total head of water on the orifice A O B, and g for acceleration of gravity.

Example 1.

I applied to the bottom of my circular reservoir of about 4 inches diameter, a conoïdal mouth-piece A B C D (Fig. 20), having nearly the form of the contracted vein issuing from an orifice in a thin plate 0·4 inch in diameter. At the small base C D, of this mouth-piece, where the diameter was only 0·313 inch, I added a conical divergent tube C D F G, $x' = 9·96$ inches long, along the axis E H, and measuring 0·319 inch diameter at the small end C D, and 0·892 inch at the large end F G,

the angle of divergence between the sides C F, D G, being therefore 3° 18′; on account, however, of the slight difference of 0·003 inch between the diameter C D, at the small base of the mouth-piece and the corresponding base of the divergent tube, the angle of divergence between the base C D of the mouth-piece, and the base F G of the tube was actually 3° 20′.

In three experiments, under pressure heads of 13·5 and 15·1 inches, I found the mean coefficient of discharge under water, through this tube, to be 2·028 at the base C D, while, with the same heads, the corresponding coefficient of discharge of the mouth-piece A B C D, alone was only ·975, on an average under water, for a head equal to, say $(2{\cdot}028)^2 \times 14$ inches$=$58 inches, whence it is clear that the discharging capability of the compound tube A B D G F C A, was 2·08 times greater than that of the mouth-piece alone.

In this instance, A $O=r=0{\cdot}2$ inch, D $E=r'={\cdot}1565$ inch, O $E=x=1{\cdot}00$ inch, E $H=x'=9{\cdot}96$ inches, F $H=r'+mx'=0{\cdot}446$ inch, $m=$tangent 1° 40′$={\cdot}029097$.

s_0 may approximately be taken at $0{\cdot}56r=0{\cdot}112$, judging by its value in other cases, and, by inspecting Tables I, II, and V, it will be seen that we can put $i_{(1)}=$ 0·41 and ·975 for (coeff. vel. orif. D E C mouth-piece.), the ratio of the theoretical velocity due to the head H, to the velocity of efflux through the orifice D E C, of the contracted mouthpiece, under a head of from 55 to 60 inches, also (coeff. velocity natural contracted vein at D E.)$=1$, nearly.

If we substitute these numbers for the symbols in the last equation and divide by $2g$ H, we find by computation 1·973 for the coefficient of discharge or velocity through the base D E C, against 2·028, by experiment.

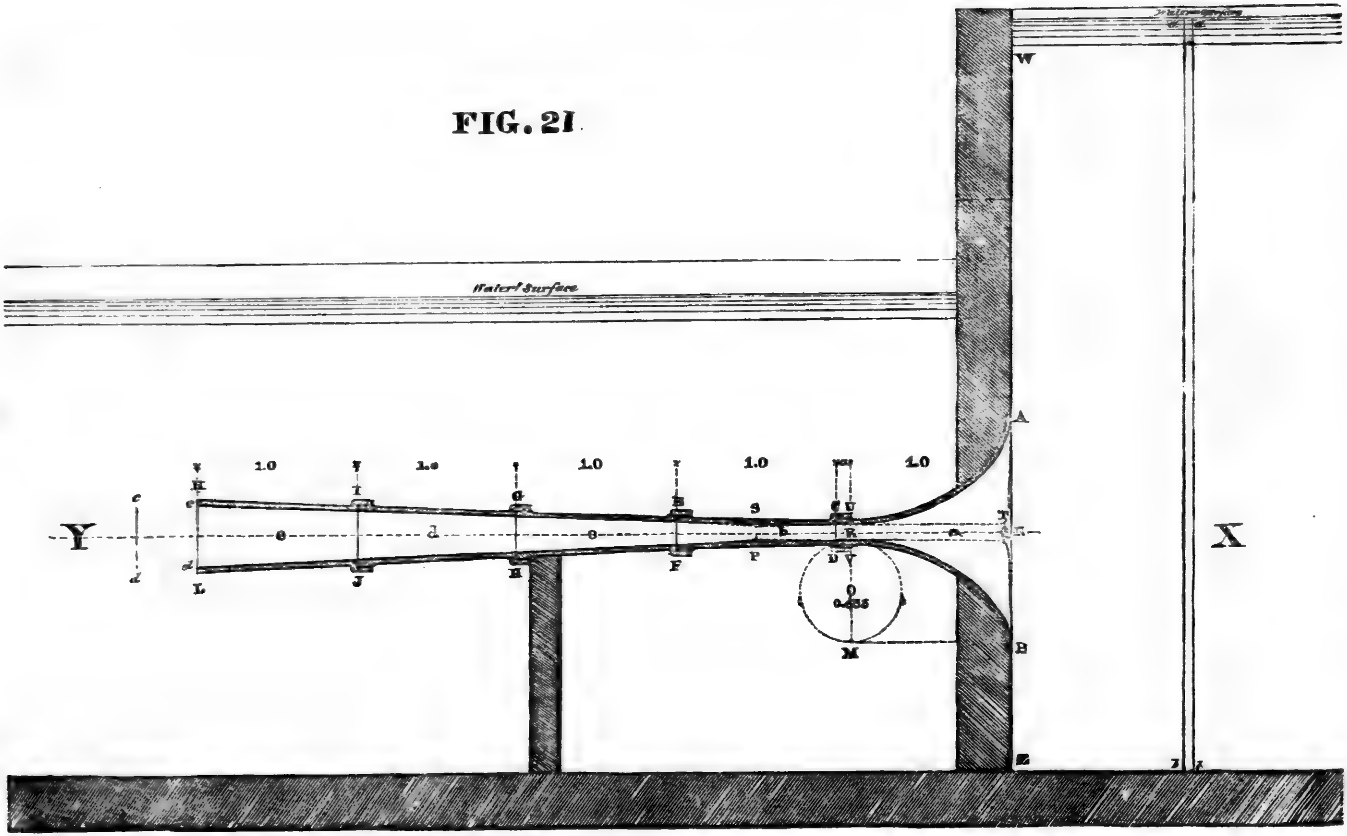
The discrepancy between the computed and the observed coefficients of discharge is probably due to an unavoidable want of accuracy in some of the factors which were introduced into the computation, and a part of the excess of the latter over the former co-efficient is also to be attributed to the great disengagement and consequent diminished mutual interference of the fluid particles moving within such tubes, in comparison to what takes place in the naturally contracted vein issuing from an orifice in a thin plate. Furthermore, the profile of the mouthpiece differed slightly from that of a perfect naturally contracted vein formed under a uniform pressure, or in the open atmosphere, the said embouchure being a little more convergent than the vein.

Example 2.

Theoretrical determination of ratio of velocity in small base of divergent tube with cycloidal mouth piece, experimented with, in 1853, at Lowell, Mass., by Mr. J. B. Francis to theoretical velocity due to head.

Mr. Francis, the celebrated American hydraulician, fitted to the vertical side W Z of a reservoir, a conoïdal mouth-piece a 1·0 foot in length from N to R, formed by the revolution of a semi-cycloid A U, generated by a point U, in a circle O, 0·635 foot in diameter, rolling along the base A M, as shown in Fig. 21, with a cylindrical prolongation U C D V, 0·1 foot long from U to C, having a diameter of 0·1017 foot between these two points. To this compound mouth-piece he joined, in a horizontal position, a divergent cast-iron tube C D L K, made in four parts b c d e, each 1 foot in length, screwed together and ground smooth inside on a mandril, with emery, but not polished, having the form of a frustum of a cone, 0·1454 foot wide at E F, and 0·4085 at K L, with sides E K, F L, containing an angle of 5° 1, joined to the cylindrical portion U C D V, of the mouth-piece by means of an arc of a circle of about 22·69 feet radius, tangent to both the conical frustum E K L F produced, and the cylinder U C D V. Although the discharge took place under water, the tube proved to be effective only for the first 3 feet, viz: up to I J, or probably for a length intermediate between 3 and 4 feet.

FIG. 21



The following characteristic results obtained are extracted from Table XXVII, of experimental data given at page 221, Lowell Hydraulic experiments, 3rd edition, 1871:

TABLE XVII½.

Nos. of experiments.	Orifice in thin plate and parts of the compound tube used. See fig: 21.	Diameter at the place of discharge. See fig: 21.	Differences of level between the water surfaces of the supplying and receiving reservoirs, or effective head H producing the discharge.	Maximum ratios of velocities at smallest section to velocities due to the heads.
		Feet.	Feet.	
94	Orifice.	0·1017	0·0916	0·5642
96	"	0·1017	0·4835	0·5797
99	"	0·1017	1·0242	0·5915
97	"	0·1017	1·4987	0·5928
2	a	C D = 0·1018	0·0340	0·8183
6	"	" — "	0·2300	0·8626
11	"	" — "	0·6590	0·9367
18	"	" — "	1·5158	0·9439
37	a b	E F = 0·1454	0·8544	1·5919
49	a b c	G H = 0.2339	1·0999	2·1643
62	a b c d	I J = 0·3209	1·1772	2·4306
78	a b c d e	K L = 0·4085	1·2823	2·4213

After making various deductions from the results of his 101 experiments on th discharge under water through the divergent tube and mouth-piece just described, Mr. Francis discusses, at pages 126, 127, 128 of his work, the application of Bernouilli's theory in connection with the large coefficients of efflux or velocity arrived at by him, as follows:

"According to Bernouilli's theory, the velocity of the water at its final discharge from the tube should be that due to the head;* in experiment 62 this velocity is 8·7018

*Call A the area of the section, and V the velocity of the water at $a\ b$ (Fig. 21), B the area of the section, and v the velocity at $c\ d$; h = the head or difference of height of the surface of the water in compartments X and Y. The motion having become permanent, we have:

$$A\,V = B\,v.$$

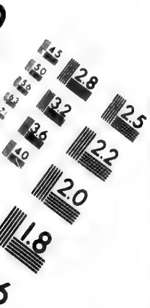
The volume of water included between the sections $a\ b$ and $c\ d$ in the small time t will move to $a'\ b'\ c'\ d'$; the volume included between the sections $a'\ b'$ and $c\ d$ is common to both positions, every particle in one having its counterpart in the other, both in position and velocity. In finding the change in the living force in the two positions, we need only consider the volumes $a\,a'\ b\,b'$ and $c\,c'\ d\,d'$. These volumes are equal, and assuming the water to be pure and at its maximum density, the weight of each is 62·382 A V t pounds.

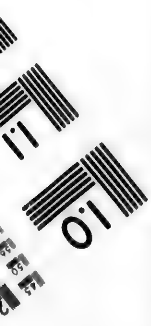
The living force of the volume $a\,a'\,b\,b'$ is $\dfrac{62{\cdot}382\ A\ V\ t}{g}\ V^2$

" " " $c\,c'\,d\,d'$ is $\dfrac{62{\cdot}382\ A\ V\ t}{g}\ v_2$

The increase of living force in passing from one position to the other being

$$\frac{62{\cdot}382\ A\ V\ t}{g}\ (v^2 - V^2) \qquad (1)$$


2.8
2.5
3.2
2.2
2.0
1.8


1.1
1.0

feet per second; the velocity at other parts of the compound tube would be inversely as the squares of the diameters; at the smallest section C D, the velocity must be greater than at the final discharge G H, in the ratio of 1 to $\left(\frac{0\cdot3209}{0\cdot1018}\right)^2 = 9\cdot9367$. To give this velocity at the smallest section without the divergent tube would require the effective head of water to be increased from 1·1772 feet to $1\cdot7772 \times (9\cdot9367)^2 =$ 116·24 feet, the increase being 115.06 feet; if the pressure of the atmosphere was great enough, its pressure, to this extent, would be rendered active. The total pressure of the atmosphere is usually about 34 feet, and this, of course, is the limit to which it can be rendered active. Abstracting from the effects of vaporization, whenever the exhausting effect of the divergent tube exceeds the pressure of the atmosphere (added to the pressure due to the actual head of water at the smallest section), breaks which must occur in the mass of water in the compound tube, at or near the smallest section, and the flow through the smallest section will be the same as if the discharge took place in a vacuum. In experiment 62, the exhausting effect of the diverging tube, according to Bernouilli's theory, exceeds three times the actual (absolute) pressure at the smallest section, and if it had produced its full effect, according to theory, or even one-third of that effect, breaks must have occurred in the mass of water near the smallest section.†"

"The ratio of the actual velocity of the water at its final discharge, to the velocity, according to Bernouilli's theory, is 0·2446, in experiment 62, or about one-quarter of the velocity due the head, indicating a loss of about $\frac{15}{16}$ of the living force. It is difficult to see how so much can be lost. There are no abrupt changes in velocity, and the interior surfaces of the mouth-piece and diverging tube are smooth and free from sensible irregularity. The slight oxidation observable after some of the experiments appears to have produced no sensible loss, as in experiment 62, which gave the greatest result, there was considerable oxidation, while in other experiments giving a less effect, there was no oxidation."

"The chief discrepancy between the hypothesis on which Bernouilli's theory is founded and the real conditions of the motion, appears to be due to the retarding effects of the walls of the tube. According to the hypothesis, the velocity in all parts of the same section is the same; Prony's well known formula for the motion of water in pipes is founded upon the idea that the principal retardation is due to the sides; whence it follows, that the velocity must be least at the sides and greatest at the centre. Darcy‡ made many experiments on the subject by means of Pitot's tube, and found that in long, straight pipes there was a material variation in the velocities at different distances from the centre, and determined a formula expressing the law of the variations. It would not be safe to apply this formula to these experiments on account of the short length and varying diameter of the compound tube, but it is clear that variations in the velocity must exist to an extent which must greatly modify the results deduced from Bernouilli's theory."

This increase of living force is produced by the action of gravity on the volume of water $A V t$ descending through the height h, which is equivalent to an amount of work represented by

$$62\cdot382\ A\ V\ t\ h. \qquad (2)$$

By the doctrine of living forces, the living force (1) is equivalent to the amount of work represented by

$$\frac{62\cdot382\ A\ V\ t}{2g}(v^2 - V^2) \qquad (3)$$

The amount of work in (2) and (3) must be equal; we have, therefore:

$$62\cdot382\ A\ V\ t\ h = \frac{62\cdot382\ A\ V\ t}{2g}(v^2 - V^2);$$

from which we deduce $h = \frac{v^2 - V^2}{2g}$

If V is very small relatively to v, it may be neglected, and we have

$$h = \frac{v^2}{2g}, \text{ and } = \sqrt{2gh}$$

†When Mr. Francis speaks of breaks occurring in the divergent stream when the exhausting effect exceeds that due to the pressure of the whole atmosphere, he, no doubt assumes, the same as Mr. Neville, that the tubes cannot run full in a vacuum.

‡Recherches expérimentales relatives au movement de l'eau dans les tuyaux, par Henry Darcy, Paris, 1857.

I suppose it is on account of the comparatively small divergence of the sides of his tube that Mr. Francis did not consider it of importance to make an allowance for the loss of head due to the variation in the element of mass moved at every instant along the path of the stream, from the smallest to the largest section of the tube, as was done in the theoretical computation of the discharge through cylindrical tubes, given at page 64, which I took from the work of Mr. J. Neville.

It was, in part, for the purpose of ascertaing approximately to what extent such losses of head may occur in tubes whose sides diverge at a small angle, that I undertook the experiments recapitulated in Table XI (page 28) on the stemming power of the naturally contracted vein in a diverging tube, under the ordinary pressure of the atmosphere.

These experiments show that a water column pressure varying from ·67 to ·71 of the pressure corresponding to the total fall from the water surface of the reservoir of supply to the orifice or inlet of the divergent tube, was accumulated in the receiving vessel before any liquid was lost or spilled laterally at the entrance of the tube. Therefore, the total loss of head caused by friction, vicosity, mutual interference, eddies and all other resistances must evidently have been less than from $(100-71) = 29$ to $(100-67) = 33$ per cent. of the total fall just referred to, while the stream was flowing from the small to the large end of the tube.

It appears, moreover, that this loss of head decreases simultaneously as the diameter of the vein and the fall from the surface of the supplying water to the orifice of the tube increase; hence, it must evidently have been less than 29 per cent. in Mr. Francis' experiment No. 62, considering that the orifice of his divergent tube was 1·22 inches instead of only 0·305 inch in my own tube, and the head of water used by him 14·1264 inches, viz.: only 1 inch less than my own fall $H_{,} = 15.15$ inches in experiment Q, Table XI.; if, however, in addition to the water colum pressure, we take into consideration the increased discharge obtained when the flow takes place in a closed, divergent, tubular envelope, the velocity head in Mr. Francis' experiment would be about 6 times as large as the fall in my experiment Q.

I think, all things considered, not much, if anything beyond $\frac{1}{4}$ of the total head of water used by Mr. Francis in his experiment No. 62, could have been lost while the stream travelled from the small to the large end of his divergent conical tube, notwithstanding that the interior conformation of this tube differed somewhat from that used by me.

The ratio of the actual velocity of the water at its final discharge from the tube, to the velocity due to this reduced head acting on the larger base of the tube is thus, in experiment No. 62, equal to $\frac{0{\cdot}2446}{\sqrt{\frac{3}{4}}} = {\cdot}2825$, the loss of living force indicated being still as large as $\frac{11}{12}$ of the whole amount.

The chief discrepancy between the hypothesis on which Bernouilli's theory is founded and the real conditions of motion in the liquid stream cannot, in my opinion, be due to the retarding effects of the walls of a conical divergent tube 0·1018 foot in diameter at the small end, having the comparatively insignificant length of 3 feet or 29 diameters, wherein the office of a large portion (if not the whole) of the capillary attraction of the very material of which the tube is formed, is to increase the velocity of the enclosed stream.

The profile of the cycloïdal mouth piece A U C D V B, having nearly 11 diameters C D in length inclusive of the cylindrical extension, which was used by Mr. Francis, apparently in imitation of Michelotti, differed much from the outline of the longitudinal section of a vein of corresponding minimum diameter C D and length as naturally formed in the atmosphere, or in any other gaseous medium under a uniform pressure or in vacuo, which is shown approximately by a dotted line in Fig. 21. By assuming that the cycloïdal mouth-piece performed the same functions as the natural conoïdal vein form just described, both when used alone and in connection with a divergent tube,* we may attempt to determine, in an approximate manner, the

*This view, however, is not strictly correct, for with a cycloidal mouth-piece the vein must continue to contract for some distance beyond the orifice C D or U V, and furthermore the pressure within the mouth-piece is necessarily variable, especially when used with the divergent tube.

numerical values of the coefficients of velocity at the small base C D. for the tubes a b, a b c and a b c d, which are respectively 2·1, 3·1 and 4·1 feet long, directly by means of formula (21) – (1°) by supposing these tubes to be nearly equivalent as regards discharging power, to tubes having true conical bores formed respectively by the revolution of trapeziums C D E F, C D G H, C D I J and C D K L, about the axis N Y – (2°) by taking for granted that their discharging power would not have been sensibly affected in any case, if instead of introducing a curved junction for the first half foot from C D, so as to avoid a sharp angle, the cylindrical portion U C D V, had been extended to meet the conical part K L F E, which junction would occur very nearly midways between E and C, or at P S = 0·50 ft., beyond C D.

According to hypothesis (1), and judging, as in previous cases, by the results given in the tables already referred to in preceding examples we may put, without risk of much error: ($i_{\left(\frac{v}{a}\right)}$ = 0·43 for the three tubes, viz.: a b, a b c, and a b c d):

$r' = \frac{C\,D}{2} = \frac{0·1017}{2} = 0·05085$ ft. = for tube a b, ·81 Q T;
for tube a b c, ·807 Q T;
for tube a b c d, ·805 Q T;

where Q T represents the radius r, of a theoretical orifice assumed to be at the point Q, situated at a distance C T = x = 1·08 ft. back from C D, whence:

r = for tube a b: $\frac{·05085}{·81} = 0·0628$ ft.,
for tube a b c: $\frac{·05085}{·807} = 0·0630$ ft,
for tube a b c d: $\frac{·05085}{·805} = 0·0631$ ft. Also,

s_0 = for tube a b: 0·57 r = 0·035796 ft.,
for tube a b c: 0·56 r = 0·03528 ft.,
for tube a b c d: 0 56 r = 0·03534 ft.,

x' = for tube a b: 1·0 ft.,
" " a b c: 2·0 ft.,
" " a b c d: 3·0.

$$\left(\begin{array}{c}\text{Coeff.}\\ \text{veloc.}\\ \text{nat.}\\ \text{cont.}\\ \text{vein}\\ \text{at}\\ \text{C D.}\end{array}\right) \times \left(\begin{array}{c}\text{Coeff.}\\ \text{veloc.}\\ \text{orif. C D,}\\ \text{mouth-}\\ \text{piece.}\end{array}\right) = \text{for tube } a\,b:\ 0·94,$$

" " $a\,b\,c$: 0·945,
" " $a\,b\,c\,d$: 0·95 for efflux under water; these last factors being taken in excess of those found by Mr. Francis for corresponding heads, as per Table XVIII½, on account of the greater efficiency of the mouth-piece for the increased velocities generated by the divergent tube.

By substituting the above values successively for the symbols in equation (21), we obtain, after dividing by $\sqrt{2\,g\,H}$, the following ratios of velocity at smallest section to velocity due to head; the tubes, as already stated, being considered as true frustums of cones, viz.:

For tube a b: 1·3606,
" " a b c: 1·8523,
" " a b c d: 2·0793.

The same ratios computed in accordance with hypothesis (b), are found to be—

For tube a b: 1·3590,
" " a b c: 1·8514,
" " a b c d: 2·0693.

These three ratios are deficient in comparison to those derived from Mr. Francis' experiments, nearly 18 per cent., for each of the three tubes. This uniform discrepancy I attribute to a supplementary conversion of acceleration into mass effected in the excessively convergent cycloïdal mouthpiece (as compared to the theoretical contraction of the natural liquid vein = $\sqrt{\frac{1}{2}}$ or ·8408), simultaneously with the reduction of the absolute pressure in the said mouth-piece through the agency of the divergent tube. In the case of the convergent mouth-piece, I conceive the process of transformation of the elements of energy to be the reverse of that obtaining in the divergent tube; in the latter the liquid is attracted by the sides, while in the

former its adherence to these sides is diminished; the pressure is, however, reduced in both instances.

Notwithstanding the unavoidable want of accuracy in some of the factors which had to be used in connection with the practical illustrations of the working of the new theory given above, it is evident that this theory leads to results much superior, in point of concordance with observed facts, to those obtained with the aid of the theories now in vogue; some of these latter results seem to me to be in direct contradiction with the actual state of matters as established by careful observations.

Discharge through Conical Convergent Tubes.

Although this class of tubes is as simple in conformation as the diverging tubes, the conditions under which the flow of liquid takes place through them, are variable not only with the degree of convergence of their sides, but also with their length.

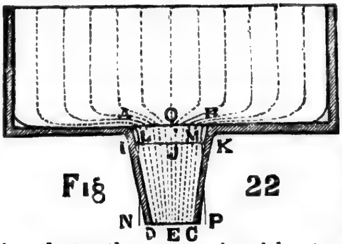


Fig 22

1st. In tubes such as A B K I A, whose sides A I, B K converge less at every point of the axis O J, than the corresponding naturally contracted vein of equal length A B M L A, projected under the same head through an orifice in a thin plate, whose area is equal to that of the large base A O B of the tube, and of a length O J $= l$ less than that for which j is a maximum, the fluid is unceasingly compelled to follow the sides A I, B K of the tube, as in the case of a plain conical divergent tube added directly to the reservoir without mouth-piece. Formula (18) is therefore directly applicable to all such tubes; the distance O J, from the orifice or large base A O B, at which the convergent tube A B C D ceases to act in a similar manner as the divergent tube, and where j is a maximum, being determined in general by the relation:

$$\frac{dj}{dx} = d\left\{\frac{2i_{\binom{v}{a}}s_0 + x + i_{\binom{v}{a}}x + 2\sqrt{i^2_{\binom{v}{a}}s_0^2 + i_{\binom{v}{a}}s_0x + xi^2_{\binom{v}{a}}s_0 + i_{\binom{v}{a}}x^2}}{i_{\binom{v}{a}}x\left\{1 + \frac{r^2}{(r+mx)^2}\right\}^2} - \frac{s_0}{x}\right\}\frac{1}{dx} =$$

$$\left\{1 + i_{\binom{v}{a}} + \frac{i_{\binom{v}{a}}s_0 + i^2_{\binom{v}{a}}s_0 + 2i_{\binom{v}{a}}x}{\sqrt{i^2_{\binom{v}{a}}s_0^2 + i_{\binom{v}{a}}s_0x + xi_{\binom{v}{a}}s_0 + i_{\binom{v}{a}}x^2}}\right\}\left\{i_{\binom{v}{a}}x(r+mx)^5 + 2i_{\binom{v}{a}}xr^2(r+mx)^3\right.$$

$$\left. + i_{\binom{v}{a}}xr^4(r+mx)\right\} - \left\{i_{\binom{v}{a}}(r+mx)^5 + 2i_{\binom{v}{a}}r^2(r+mx)^3 - 4mi_{\binom{v}{a}}xr^2(r+mx)^2 + i_{\binom{v}{a}}r^4\right.$$

$$\left.\times(r+mx) - 4mi_{\binom{v}{a}}xr^4\right\}\left\{2i_{\binom{v}{a}}s_0 + x + i_{\binom{v}{a}}x + 2\sqrt{i^2_{\binom{v}{a}}s_0^2 + i_{\binom{v}{a}}s_0x + xi^2_{\binom{v}{a}}s_0 + i_{\binom{v}{a}}x^2}\right\}$$

$$-\left\{s_0 \text{ hyp. log. } x\right\}\left\{r+mx\right\}^5 = o \qquad (22)$$

I know of no experiment made with tubes so conditioned.

2nd. When the sides A D, B C, of a tuble A B C D, converge at every point more than the corresponding outside portion of the naturally contracted vein A L N P M B, projected through an orifice in a thin plate equal to the large base A O B of the tube, or when they converge less than this naturally contracted or theoretical vein only for a part O J of its length, as in the tube A B C D, and for the remainder J E of the distance O E, from the large base A O B to the small base D E C, more than said portion of contracted vein A B P N A, it is clear that here also the same as in divergent tubes, motion assumes a permanent state in the tube taken as a whole, only after the initial fluid sheet occupying the plane A O B has passed the section D E C, contrary to what takes place in the naturally con-

tracted vein, in which the conditions of motion in the posterior portion A B M L A can evidently not be affected by any change that may take place in those of the fluid particles passing at D E C.

In all such tubes, any difference existing between the velocity of the fluid issuing from the tube at the small base D E C, and that of the naturally contracted vein A B M P N L A, at the corresponding section N D E C P, is the result of an artificially increased, or partly increased and partly diminished velocity, due to the force fi_c, viz., of that corresponding to $\sqrt{i_{\binom{v}{a}}s_0 + x}$ in the said natural vein. This transformed velocity may, in general, be represented by $\sqrt{i_{\binom{v}{a}}s_0 + jx}$, where j is a number greater than unity for increased velocities, and less than 1 for diminished motion, or rate of progress of the vein, as regards the force fi_c. The expression: $\dfrac{\sqrt{i_{\binom{v}{a}}s_0 + x}}{\sqrt{i_{\binom{v}{a}}s_0 + i_{\binom{v}{a}}x}}$, which, as already stated and explained, represents, in general, the velocity ratio v_p of the motions due to the forces fi_c and fi_o, at any point of the naturally contracted horizontal vein outside of the reservoir, is converted in the convergent tube A B C D, into:

$$\frac{\sqrt{i_{\binom{v}{a}}s_0 + jx}}{\sqrt{i_{\binom{v}{a}}s_0 + i_{\binom{v}{a}}x} + \sqrt{i_{\binom{v}{a}}s_0 + x} - \sqrt{i_{\binom{v}{a}}s_0 + jx}}$$

and the same as for conical divergent tubes, we may put:

$$\frac{\sqrt{i_{\binom{v}{a}}s_0 + jx}}{\sqrt{i_{\binom{v}{a}}s_0 + i_{\binom{v}{a}}x} + \sqrt{i_{\binom{v}{a}}s_0 + x} - \sqrt{i_{\binom{v}{a}}s_0 + jx}} = \frac{r^2}{(r - mx)^2}$$

where r stands for the radius A O = O B, and m for the tangent of half the angle of convergence; whence we deduce:

$$j = \frac{\left\{\frac{r^2}{(r-m)^2}\left(\sqrt{i_{\binom{v}{a}}s_0 + i_{\binom{v}{a}}x} + \sqrt{i_{\binom{v}{a}}s_0 + x}\right)\right\}^2 - i_{\binom{v}{a}}s_0}{x\left(1 + \frac{r^2}{(r-m)^2}\right)^2} \tag{23}$$

If, now, we substitute this value of j in the expression:

$$\frac{\sqrt{i_{\binom{v}{a}}s_0 + i_{\binom{v}{a}}x} - \sqrt{i_{\binom{v}{a}}s_0 + jx} + \sqrt{i_{\binom{v}{a}}s_0 + x}}{\sqrt{i_{\binom{v}{a}}s_0 + i_{\binom{v}{a}}x}}$$

which indicates the relation between the absolute number of particles that pass in an orifice in a thin plate having a diameter equal to A B, and those passing at he large base A B, of the convergent tube, we obtain:

$$\begin{pmatrix}\text{coeff.}\\ \text{vel}\\ \text{large}\\ \text{base A B}\\ \text{convergt}\\ \text{tube.}\end{pmatrix} = \begin{pmatrix}\text{coeff.}\\ \text{vel}\\ \text{oriff A B}\\ \text{in thin}\\ \text{plate.}\end{pmatrix} \times \tag{24}$$

$$\left[\frac{\sqrt{i_{\binom{v}{a}}s_0 + i_{\binom{v}{a}}l} - \sqrt{i_{\binom{v}{a}}s_0 + \dfrac{\left\{\frac{r^2}{(r-ml)^2}\left(\sqrt{i_{\binom{v}{a}}s_0 + i_{\binom{v}{a}}l} + \sqrt{i_{\binom{v}{a}}s_0 + l}\right)\right\}^2 - i_{\binom{v}{a}}s_0}{\left(1 + \frac{r^2}{(r-ml)^2}\right)^2}} + \sqrt{i_{\binom{v}{a}}s_0 + l}}{\sqrt{i_{\binom{v}{a}}s + i_{\binom{v}{a}}l}}\right]$$

where l is substituted for $x = $ O E, the length of tube.

. Without a thorough knowledge of the laws governing the variations of $i_{\binom{v}{a}}$ and s_0, it is impracticable to determine accurately, by computation, the velocity at the small base C D of the tube.

Moreover, on account of the sharp turn of the liquid fillets about the angle of the junction of the tube and reservoir, it is probable that these do not adhere to the sides of the tube before striking against the same, wherefore a part of the efficiency assumed for the tube in constructing formula No. 24 is lost, and the discharge is also affected by friction.

The approximate determination of the coefficient of efflux for one of the conically convergent tubes, experimented with by Messrs. D'Aubuisson and Castel, referred to hereunder, was undertaken chiefly for the purpose of showing that the above formulæ lead in the right direction.

With a tube 1·767 inch in diameter at the large end A B (Fig. 22), 0·61 inch at the small end C D, having a length E O = 1·575 inch = nearly 2·6 diameters of the small base and sides A C, B D, inclined at an angle of 40°, 20′, the coefficient of efflux for the small end was found, by experiment, to be 0·87 under a head of 9·84 feet.

Putting, in this case: $i_{\binom{v}{a}} = {\cdot}47$, $s_0 = 0{\cdot}6$ inch and $\begin{pmatrix}\text{coeff}\\ \text{vel}\\ \text{orif}\\ \text{A B}\end{pmatrix} = 0{\cdot}62$; also, $r = 0{\cdot}8835$ inch, $l = 1{\cdot}575$ inch and $m = \text{tang } 20°, 10' = 0{\cdot}36726$. We obtain, by using formula (24):

$$\begin{pmatrix}\text{coeff}\\ \text{vel}\\ \text{base}\\ \text{A B}\\ \text{cony}\\ \text{tube}\end{pmatrix} = 0{\cdot}1154 \text{ and } \begin{pmatrix}\text{coeff}\\ \text{vel}\\ \text{base}\\ \text{C D}\\ \text{cony}\\ \text{tube}\end{pmatrix} = \begin{pmatrix}\text{coeff}\\ \text{vel}\\ \text{base}\\ \text{A B}\\ \text{cony}\\ \text{tube}\end{pmatrix} \times \frac{1{\cdot}767^2}{0{\cdot}61^2} = 0{\cdot}9686.$$

On the Flow of Liquids through Oblong Orifices in Thin Plates.

Numerous experiments were made by Messrs. Poncelet and Lesbros, at Metz, in 1826 and 1827, upon efflux through large rectangular orifices, pierced in a vertical brass plate 0·1575 inch thick, so as to obtain a perfect contraction of the stream. The widths of these apertures were generally 7·8737 inches, and in some cases 23·6211 inches, while their heights varied from 0·3937 inch to 7·837 inches.

Although these experiments are, with good reason, considered to be the most accurate available for practical purposes, on account of the uncertainty, as regards the effective head and nature of the contraction of the vein, arising from the fact of a depression taking place during efflux, in the water surface of the supplying reservoir, immediately behind the vertical side or partition which contains the orifice, they are obviously not suitable for use in connection with theoretical investigations.

The only experiments I know of which appear to me to have been made in the proper conditions and with the requisite amount of care, to be serviceable for theoretical purposes, are those by Messrs. Castel and D'Aubuisson de Voisins, with rectangular orifices 0·328 feet = 3·936 inches long and 0·033 feet = 0·399 inch high, pierced in a vertical partition; the ratio of the length to the breadth being, therefore, equal to 9 9398. The mean results obtained by these engineers are given in the following table:

TABLE XVIII.

Number.	h Depth of upper side of orifice below water surface.	H Depth of lower side of orifice below water surface.	$D = \frac{2}{3} c_d l \sqrt{2g}(H^{\frac{3}{2}} - h^{\frac{3}{2}})$ = Discharge per second.	c_d Coefficient of discharge or velocity, the theoretical velocity due to the mean pressure of $\frac{2}{3}\left(\frac{H^{\frac{3}{2}} - h^{\frac{3}{2}}}{H - h}\right)$ on the orifice being equal to unity or 1.
	feet.	feet.	cubic feet.	
1	0·0491	0·0821	0·01607	0·728
2	0·0819	0·1149	0·1946	0·720
3	0·1147	0·1477	0·2242	0·719
4	0·1475	0·1805	0·2497	0·715
5	0·1804	0·2134	0·2723	0·710

In common with the last-named and other experimenters with oblong rectangular orifices and the like, I found, under a small head of about 3 inches, that the coefficients of efflux or velocity proper to annular and lunular-shaped orifices, are invariably greater than those corresponding to orifices which embrace the full area enclosed within the circumference of a circle.

1. When ratio between the breadth and the mean length of the annular space or opening formed by introducing a cylindrical rod, 0·185 inch diameter, in the reservoir opposite an orifice in a thin plate 0·4 inch diameter, was 8·55, the coefficient of discharge was about 0·7256, with the base of the cylinder in the plane of the orifice; this coefficient became, however, reduced to 0·68, when the cylinder protruded through the plate 0·2 inch beyond the plane of the orifice, as shown in table VI.

2. When this ratio was increased to 20·70, by introducing into an orifice 0·482 inch diameter, a disk 0·355 inch diameter and 0·048 inch thick, the coefficient of discharge rose to 0.7948 for the upper base of the disk in the plane of the orifice, and to 0·8098 for the lower base in the plane of this orifice, as per Table VIII.

When the ratio between the mean length and breadth of the ring-shaped aperture was still further augmented to 80·35, by introducing the disk just described into an orifice 0·384 inch diameter, the coefficient of discharge rose as high as 0·8907 for the lower base of the disk in the plane of the orifice, and 0·91 for the upper base in this plane, as per Table IX.

4. When the discharge took place through the lunular-shaped opening left between the circumference of a cylindrical rod 0·185 inch diameter and that of an orifice 0·4 inch diameter, as shown in the figure at the head of Table VII, the coefficient of discharge was 0·7016 while the base of the cylinder coincided with the plane of the orifice, and about 0·663 when the rod projected 0·2 inch below this plane.

In all these experiments of mine, however, the contraction was probably modified, and, to a small extent, destroyed along the longitudinal face of the rod or disk introduced into the reservoir and let down below the plane of the orifice, for which reason the discharge proved, perhaps, slightly larger in each case than it would have been, if the stream had been allowed to contract freely all around the perimeter of the orifice.

If the larger coefficients, obtained in the four cases just referred to, are corrected for this want of completeness of the contraction of the stream—approximately

in accordance with the empiric rules given by some authors, they become reduced, respectively, from 0·7256, 0·8031, 0·91 and 0·7016, to about 0·700, 0·77, 0·85 and 0·68; they remain much higher, however, in any case, than the coefficients which are proper to a circular orifice of equal area for efflux under the same head.

There is no apparent reason why the first slice or sheet of liquid leaving the orifice at the instant it is opened, should move off faster, under the same pressure, from an oblong than from a perfectly round or circular orifice in a thin plate, and I see no other cause for the increased discharge obtained than the following:

When one elementary slice or sheet of liquid of the oblong-shaped vein tends to detach itself from the next succeeding one, and that, owing to the intermittent action of the resistance or force of cohesion, the motion of the liquid particles, or fillets, becomes accelerated, and consequently the total area of the moving stream correspondingly diminished, the increased rate of contraction in the direction of the longest of the radii, which extend from the perimeter of the oblong orifice or vein to the centre of figure, as compared to that taking place along the shorter radii, produces, together with a change of form, also an enlargement of the sectional area embraced by the spurting liquid vein through the admixture of air with the water or otherwise—when the conditions of flow become similar to those of divergent tubes.

Liquid Pressure, Motion, Energy, &c.

Pressure is most frequently generated in liquids, whether in a state of rest or in motion, by gravity acting on a large number of particles superimposed to one another; but it also often results from the action of a piston moved by some exterior force. No matter how generated, it may be considered in the light of an artificial increase, in the natural force of repulsion co-existing with that of attraction between all molecules.

When the force of attraction is artificially increased, instead of that of repulsion, the result is the opposite of pressure, viz., dilatation or distention or exhaustion.

Liquid motion and energy are, in all cases, governed by the differences between the forces of attraction and repulsion obtaining at the origin and along the path of the stream.

If a pressure p, has to be applied during the small space of time $d\,t$, in order that a liquid particle may describe, within the sphere of molecular oscillations, the small distance $d\,x$, necessary to overcome the force of cohesion, together with the inertia of the said particle—according to the laws of uniformly accelerated motion—another pressure $n\,p$, will have to act during a length of time $= d\,t\,\frac{\sqrt{p}}{\sqrt{n\,p}}$ to cause the same particle to describe the distance $d\,x$, that is to say, the number of times which one and the same space $d\,x$, is passed over in the unit of time, say one second, by successive molecules, varies as $\surd$, of the intensity of the pressure to which the particle is subjected.

In the case of a liquid vein issuing from an orifice in a reservoir by virtue of the action of gravity alone, the absolute velocity varies therefore, as the $\surd$ of the depth of the centre of pressure on the orifice below the surface, being theoretically equal to $0.7071 = \sqrt{\tfrac{1}{2}}$ of that attained by a body after having descended freely through a space equal to the said depth, wherefore, abstracting all causes of incidental perturbation, the energy of such a vein is directly proportional to the pressure or head on the orifice.

This constitutes the basis of the generation of the absolute velocity and energy proper to a liquid vein taken as a whole, thus: if a circular vein having a mean diameter of say 1 inch between two points, A, B 1 foot apart, of its trajectory, and formed under a water column pressure of 1 foot, takes say $\frac{1}{8}$ of a second to travel freely from A to B, another vein of the same dimensions betweeen the said points, but

generated by a hydrostatic pressure of 4 feet, yet in every other respect formed under the same conditions as the first jet, will fill up the space of 1 foot, referred to between A and B in $\frac{\frac{1}{8}}{\sqrt{4}} = \frac{1}{16}$ second :—wherefore the quantity of water supplied by vein No. 1 will bear to that afforded by vein No. 2 the ratio of 1 to 2, and energy will be developed in the ratio of 1 to 4.

The absolute rate of motion or velocity just referred to, which is proper to the whole of the elementary liquid slices of which every jet may be conceived to consist, is quite distinct, however, from the rate of progress of one and the same elementary sheet of liquid in assuming different positions successively along the path of the stream. It is by this relative motion or rate of advance, that the outline of the conoïdal space swept out by the contracted vein and the distribution of pressure in tubes are essentially controlled. The relative velocities of an elementary volume of liquid ejected from the reservoir corresponding to the area of the orifice, are governed by the ——— elementary impulses or increments of acceleration which are imparted, in rapid succession, to the increment of vein considered, from a state of rest all along its trajectory ; these impulses having to overcome alternately cohesion and inertia combined and ~~a reduced inertia alone.—as already~~ explained in another part of the paper.

In the naturally contracted vein the pressure is null, or *o*—from the theoretical orifice, which is situated at the plane, where the total acceleration or velocity, generated by the ——— small impulses ~~applied~~ against cohesion and inertia combined, is equal to the velocity due to the impulses expended in overcoming a reduced – inertia alone—to the end of the vein outside the reservoir ; from the said orifice to the plane of rest, the pressure gradually increases, becoming equal to that due to the full head at the said plane.

When a divergent tube is added to a conoïdal mouth-piece, having the form of the naturally contracted vein, the molecular force of attraction is increased so as to produce a dilatation or distention in the liquid filling the mouth-piece, which probably diminishes in intensity, from the smallest section to the theoretical orifice, and thence to the plane of rest, where the full hydrostatic pressure again obtains. In the divergent tube itself, the exhaustion decreases gradually from the small to the large base, where it is reduced to a minimum. Thus, if the total velocity generated, by the addition of the divergent tube, at the smallest section, is to that obtained at the same place with the mouth-piece alone, in the ratio of 2 to 1, the force of attraction will be increased by a quantity equal to $2^2-1=3$ times the pressure due to the head of water on the centre of pressure of the section of the tube.

If the same divergent tube was added directly to the reservoir, viz., without the intervention of a conoïdal mouth-piece, the force of attraction would also be increased, but to a less extent.

In a conically convergent tube, or over-convergent mouth-piece, of any description, added to the side or bottom of a reservoir, with or without natural conoïdal mouth-piece, the force of repulsion or pressure diminishes during the flow of the liquid from the large towards the small base. In order that the whole volume of liquid may pass at the large base, which can be ejected through an orifice having an equal diameter, by virtue of the pressure in the reservoir, the force of attraction must be increased in the same manner as in the divergent tube, and *vice versa*, if the force of attraction is increased at the small base of a convergent tube, by the addition of a divergent tube, the discharging power of the former and of the two tubes combined is increased as compared to the power of a natural conoïdal mouth-piece, having its orifice at the small end equal to the small base of the convergent tube.

CONCLUDING REMARKS.

It was in the year 1645 that the Italian mathematician, Toricelli, enunciated the theorem which bears his name and may be stated as follows :—

" Generally and making abstraction of every obstacle or all cause of perturbation, " the velocity of a fluid at its passage through an orifice made in the side of a

"reservoir, is the same as a heavy body would acquire in falling freely from the "height comprised between the level of the fluid surface in the reservoir and the "centre of that orifice."

About the year 1738, Daniel Bernouilli propounded his theory, viz.:—"At any "point of a system of hydraulic conduits or pipes, the absolute total head or pressure "is composed of the pressure of the atmosphere, the actual hydrostatic pressure or "head, the head due to the velocity of the water and the head consumed by friction "and other resistances encountered between the water surface of the source of supply "and the point considered.

Ever since, it would appear to have been the constant aim of all hydrodynamicians to determine the nature and intensity of the resistances to be overcome under all possible conditions, by making numbers of experiments varied in a thousand ways, from which empirical coefficients of friction, contraction, velocity and efflux could be deduced and formulas based thereon.

If, despite all the labours and pains taken by eminent men of all ages to place the science of hydraulics on a solid basis, there is still room for much improvement, judging by the discrepancies which exist between experimental data of apparently similar nature, furnished by different authors and the variations in the formulas given in works which are all held in equally high estimation, as also by the failure of water works systems to prove equal to the requirements of the services which they were calculated to perform, it must be attributed, I think, to the fact of no one having apparently thought it necessary to take into account, independently of all resistance caused by friction, sharp curves, sudden enlargements, etc., the influence of the force of cohesion or aggregation which unites the fluid molecules into one homogeneous mass, and prevents their isolation.

If I have alluded to the shortcomings of the theories advanced and of some of the experiments made by the learned authors whose names are mentioned and others, it is certainly not with any intention of making disparaging remarks respecting the arduous labours performed by them, but solely as a means of assisting in the advancement of a science the principles of which are still imperfectly understood, and, in hopes of attracting men of science, endowed with greater powers of penetration, and more generously favoured, as regards spare funds and time, than I am, to consider the suggestions thrown out herein with a view of placing the theory of hydraulics on a sounder basis.

APPENDIX.

PHISICO-MATHEMATICAL THEORY OF THE MOTION OF LIQUIDS ISSUING FROM ORIFICES IN RESERVOIRS, BY MR. LE CHEVALIER LORGNA.

INTRODUCTION.

It is not to be denied that certain parts of natural philosophy owe everything, so to speak, to the mathematical sciences—and that other parts are much indebted to them, for, these sciences have fortunately rendered tractable things, into which neither reason nor experience, alone or combined, would ever have been able to penetrate so far. But in a great number of other instances these sciences have really not been of any assistance towards making a forward step; unless we are prepared to accept, in the case of natural things, that which will never be, viz.: the truths of computation for truths of fact, but which has taken place to a singular extent in those instances where the character and conditions of the object are totally changed when

by abstraction, it is stripped of everything that constitutes it—as nature demands that it should be, in the structure of the world.

In point of fact there is not, for example, on the intimate affections and motions of compressible and incompressible fluids, a theory founded chiefly on mathematical principles which, as might happen in mathematical philosophy, could lay claim also with an equal right and above all exception, to a place in the natural science of nature.

And if such means of investigation were to fail us, what other course would there be at our disposal for penetrating deeply into the study of this science, considering that the constituent principles of the objects are unknown to us and that the various characteristic properties are closely interwoven with very obscure and imperceptible forces.

If I do not mistake, the mode of proceeding which seems most appropriate is that of very attentive observation and reasoning, making a judicious use of one and the other by the methods of decomposition and composition—to wit, by the methods of analysis and geometry and by profiting also, in case of need, of the symbols of the one, and the figures of the other, but invariably as instruments only, and when the things or their parts can, without being disfigured, assume the character of simple homogeneous quantities, be subordinated to mutual relations, and even be represented to the senses, under the abstract figures of geometry.

Would not that be the true use of mathematics in connection with natural philosophy? It is not meant that all suppositions are excluded from this manner of philosophising; it is sufficient that such assumptions be reasonable and reasonably admissible in physics—as the postulates are in geometry, and not ideal and arbitrary or made for the sole purpose of adapting the object to the laws of computation.

No doubt, this method is not that which is most followed, because it is not the most accepted, nor the easiest—and that it is much more convenient and pleasing to human pride to pretend having found than to find out actually what nature performs. It is for this reason that Mr. D'Aembert has not hesitated to declare that now-a-days every thing is accomplished by means of suppositions and computations. However, that may be, if it is not the simplest, this method is undeniably the surest and it leads to the truth, or at least to results which are not very far removed from the truth and which time does not obliterate so easily as it obliterates inexorably our comments It is upon these principles that I have undertaken and effected this investigation, as by trial, and as well as it lay in my power—of the motion of liquids within and outside of the reservoirs where they are maintained at a constant level during the flow.

The principal properties which distinguish the liquids from any other known fluid, to wit: natural incompressibility, perfect mobility and the very strong affinity of aggregation commonly called reciprocal adherence of molecules—exert an influence on their affections, that without having a particular regard for these properties as indicated by the phenomena, we can never hope to attain sound knowledge as regards the very complex irregularities of their motions. The only time, it appears when we may dispense considering these properties, which are the cause of particular actions taking place among the molecules, one upon the other, is when none of them are disturbing the general movement; in this circumstance it is permitted to view the liquid in the light of an imperfect fluid and to subordinate it in a manner to the laws of dynamics.

In such case, for example, I have thought a liquid vein in motion could be imagined to be established whose molecules are continously urged on with a uniform velocity in one and the same direction; and by this means I have endeavoured, in another paper which will be found in this volume, to bring under the dynamical laws, the appreciation of the permanent impulsion of liquids against plane surfaces. But in every other condition of things, if the properties enunciated exert an essential influence on the phenomena, it will be necessary, in order that the theory may not be wrong, that it should always be based on facts and that it should invariably be directed in the path pointed out to us by these experiments alone wherein liquids have acted naturally and such as nature has constituted them.

I do not know if I have succeeded in my undertaking, as was my intention, but in any case the failure will be due to my want of ability and not to the method which I have laid down for my guidance.

CHAPTER I.

NATURAL PHENONENA.

I.

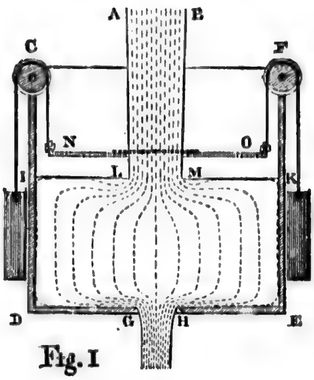


Fig. I

Phenomenon 1.—If a perennial vein of water A B, (Fig. 1) flows into a reservoir placed underneath and having any form whatever C D E F, in which the orifice of the bottom G H, where the incoming water is to escape, is smaller than the area of the cross-section of the vein A B, *it will be noticed* that a certain quantity of water is first spilled and spreads over the closed bottom G D, H E, and then, after a certain time, the liquid assumes a height such as D I, above the bottom, the surface being continually agitated by the influx of the vein, and once the efflux is equalized with the influx, the water-level I K, remains stationary, as long as the same conditions continue to subsist; nevertheless, the flow here is interrupted in the direction of the vein at L M, and continues its course until after the liquid issues from the orifice G H.

II.

Phenomenon 2.—And if several openings, smaller or greater than G H, are pierced in thin plates of metal, which can be applied to the bottom D E, it will be remarked, that by using openings getting smaller and smaller, the surface I K, is formed and maintained at a level more and more elevated above the fixed bottom D E; on the contrary, by applying successively to the bottom, orifices getting greater and greater than G H, the permanent height D I, of the water above the bottom diminishes more and more, and even disappears entirely when the vein A B, flows freely past the bottom D E.

III.

Phenomenon 3.—But, if the inflowing vein is received in a recipient N O, placed quite close above the surface I K, pierced by small holes, so that the water may descend in very small fillets, it will be seen that the surface I K, remains sensibly horizontal during the flow, as if the body of liquid I D E K, was stagnant.

IV.

Corollary I.—It is therefore evident that the liquid spilled and spread on the bottom D E, is an over-flowing liquid.

Corollary II.—And that the surface I K, is the limit of the over-flowing.

Corollary III.—And, as on the one hand, the sensible horizontality of the surface I K, during the flow, and on the other, the successive flowing necessary to supply the quantity discharged through the orifice G H, give rise to a sensible state of rest on one side and of motion on the other, the undisputable result of these phenomena is, that the condition of the flowage I D E K, is a certain singular state which participates both of rest and of motion, and which is, consequently quite distinct from the absolute state of either rest or motion.

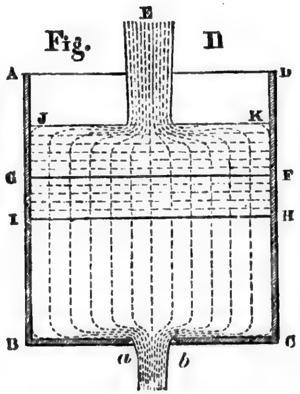
V.

Scholium.—We shall see hereafter how these few certain phenomena, which are the real axioms of natural philosophy, enlighten reason and guide it in finding out possibly the properties of liquids issuing from orifices in reservoirs, when the water contained therein is maintained at a constant height above the level of the orifices. It is a decisive step in this very obscure matter, to have discovered, as we shall see, that the state of the liquids in the interior of vessels is in a state of overflowing and that this state is mixed and distinct from that of rest and motion taken in their absolute sense, but participating nevertheless of both.

But, before going any further, let us examine other phenomena which will show us more manifestly what is the use of these flowages, by moving their limits further and further away from the orifices of the vessels, while expelling the liquids successively through smaller and smaller orifices.

VI.

Fig. II



Phenomenon 4.—Let a glass recipent A B C D (Fig. II) be prepared, in the bottom of which an opening *a b*, is made. Let a perennial vein greater than the opening *a b*, continually throw into this vessel, during a given time, a given quantity of water, and let the the water held back overflow into the vessel up to the elevation B G, there to assume the horizontal surface F G, the limit F G of the flowage being marked carefully on the glass.

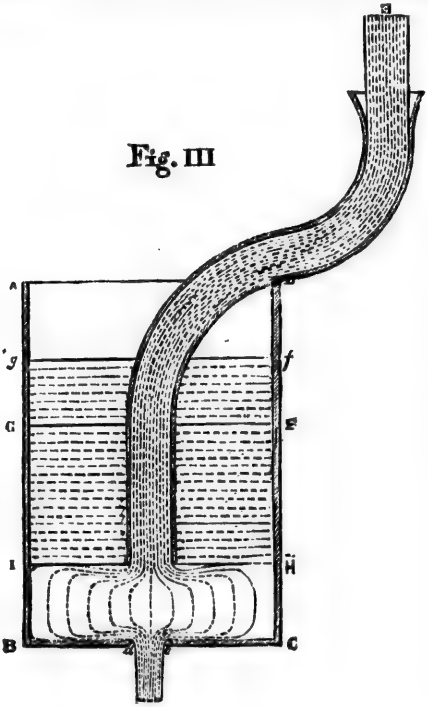
This being done, let the vessel A B C D be removed from under the vein E, and after having let out some of the liquid through the opening *a b*, let this orifice be closed, and in place of the water wasted let about an equal quantity of common oil be introduced. After this let us bring the vessel again under the vein E, so that the water may fall on the oil and traverse it to arrive at the surface of the water lying below; then let the orifice *a b*, be opened anew. After the lapse of a certain space of time during which one can see the oil ascend and descend alternately, it will be noticed: 1. That the surface of the oil stands still a little above the limit G F, indicated by the water, and the efflux of the water through the orifice becomes again permanent, as before the introduction of the oil. 2. That the surface of the water below the oil assumes and retains constantly a sensibly horizontal position, such as H I. 3. That the water introduced traverses the oil as if passing through a filter and enters the body of water underneath without producing any undulation therein, merely compensating for the discharge through the orifice *a b*, under the head afforded by the two heterogeneous liquids, as

was the case under the permanent height B G, of the originally overflowed homogeneous liquid.

VII.

Phen. 5.—If this experiment is repeated with an increased quantity of oil, it will be seen that the surface of the water lying underneath can be lowered permanently nearly down to the plane of the orifice *a b*, especially if this orifice is very small. Yet the oil continues to rest on top of the water, internally troubled on account of its affording a passage to the water which goes to compensate for the discharge, but remaining, with respect to itself, as a mass of stagnant liquid; and, moreover, it will be remarked that the greater the quantity of oil poured upon the water the more will the surface of the oil rise above the level J K, originally established.

Fig. III



Phen. 6.—If another apparatus is used, and instead of being introduced immediately into the oil—as in Fig. II, if the water of the perennial vein E be conducted separately through a pipe down to the surface I H, of the body of water lying below the oil, as shown in Fig. III, it will be observed that as soon as the flow becomes permanent: 1. The oil on top of the water remains motionless, as if it was a solid body, even though the surface of the water should descend close to B C—which is admirable to behold. 2. The surface *f g*, of the oil remains, as before, above the level F G,—assumed by the overflown water, and yet a little higher, on account of the volume of oil displaced by the pipe kept immersed therein during the efflux. 3. Finally, if, in the various attempts, an account, as correct as the circumstances will permit, is kept, whether of the quantity of oil introduced or of the quantity of water expelled from the vessel by the oil poured in it, it is found, if I made no errors in taking these very delicate measurements, that after the permanent flow is established, the weight of the oil introduced remains constantly a little greater than that of the water expelled; this, it has appeared to me, should be attributed to the adherence of the oil to the sides of the vessel, owing to which adherence the pressure of the oil on the water underneath is somewhat diminished.

IX.

Corollary I.—In the meantime, it remains decided by those phenomena that the velocity of the water issuing from the opening *a b*, cannot, at all, be due, as was

thought by Newton, to the actual descent of the liquid from its permanent surface F G, to the plane of this same orifice *a b*—whilst it might also be due to the various other falls from different other elevations, such as I H (Figs. II and III), which is absurd; considering that there is a downward motion only in the water at the bottom of the vessel, and none whatever if the superimposed oil was substituted for water—which oil is quiescent and fixed in its position during the flow.

Corollary II.—And because the oil acts on the water lying below it, per *modum unius* (as a whole), like a loaded piston pressing upon the surface I H, of the water, it is evident that the pressure exerted around the opening (*pressione circumfusa alforo*) is not merely that of the perpendicular column, having this same orifice for a base, as was thought by M. M. Varignan, Hermann and many others, but, indeed, that of the whole body of liquid. For, since it is possible to bring down the surface I H, of the water, more and more towards the bottom, simply by increasing the height of the superimposed oil, and keeping up a uniform efflux by the introduction of water through a pipe or tube, as above indicated—and considering that the oil never acts otherwise than a piston, exerting an equal pressure on all points of the infinite section I H—it follows that the action of any column, whatsoever, of definite dimensions, is not possible, nor can a determinate descent or fall take place, as was demonstrated in the preceding corollary.

Corollary III.—It is proven by the phenomena that the water maintained within reservoirs, at a uniform height above discharging orifices, is an overflown liquid and that in this overflown state, the pressure exerted by the mass of liquid around the opening acts like a piston to eject the water through this opening, and that consequently, the force which the water has at its exit from the vessel has not, any more, been imparted to it, by virtue of its actual descent or fall from the surface or limit of the overflow to the opening, than it has been produced merely by the pressure of the vertical fluid column having the opening for its base.

We can, therefore, plainly understand why the limit of the overflow rises the more above the level of the orifice, as this orifice diminishes in area—and that it falls lower and lower as the orifice is being enlarged, vanishing entirely, together with the overflow itself, when the supplying vein passes freely through the opening.

Corollary IV.—Furthermore, we can now clearly understand in what manner acts the sensible rest existing within overflown liquids, about the sensibly horizontal position of whose surface or limit of overflow there is no more any doubt, considering that it is principally the pressure which urges on the liquid towards the orifice. And this kind of downward movement of the liquid which, nevertheless, takes place in this overflown state, appears clearly to be but the successive reflux of the molecules towards the orifice on account of the successive compensating substitution of water for the water which flows out, this being a reflux which must make itself felt throughout from the bottom to the upper limit of the overflown liquid owing to the very delicate yieldingness of its parts, without actually expelling the water through the orifice. As to the manner in which subsist and are verified sensible rest in a body of overflown liquid and an interior downward motion having no part in the production of the flow in the orifices, it will supply the argument of another special exposition which will be made further on.

X.

Phenomenon 7. If once the flow from the glass receiver A B C D (Fig II), which contains nothing but overflown water up to the level F G—has become permanent, small pieces of Spanish wax, or of some other similar body slightly heavier than water, are dropped into the vessel along its sides, we observe that the small pieces of solid matter descend slowly towards the bottom in a nearly vertical direction—until having reached a point very close to this bottom—their path becomes visibly inclined and curved towards the opening, and when making their exit they all pass close to the edge of this orifice, forming a sensible determinate acute angle with the bottom. This phenomenon has been first observed by Mr. Daniel Bernouilli, afterwards by the "Abbe" Bossut, as may be seen in their excellent treatises of hydrodynamics, and I have punctually repeated and verified this observation last year.

XI.

Phenomenon 8.—Having gathered and measured the quantity of water which passed under different permanent heads in the reservoirs, through the orifices, whether pierced in thin plates or provided with additional tubes, it has thus been found that in all the experiments made by the most careful and trusty experimenters—the velocities acquired by one and the same fluid issuing through the same tube or orifice pierced in a thin plate—bear to each other the sub-duplicate ratio of the permanent heights of the fluid above the centre of the orifice. The more recent observations, viz., those which, through Royal munificience are just after being instituted on a grand scale at Turin (*Michelotti, Sper. Idraulia, e mem. dell' Ac. R. per gli anni* 1784-85), concur with all the observations made in bygone times in proving the truth of this law, so that there is perhaps not a single natural phenomenon so constantly established as this one.

Corollary —Therefore, from whatever elevations a heavy body at rest may descend freely, it can acquire, at the end of the motion, the actual velocities of the water issuing from the same orifice under different permanent heights of liquid in the reservoir, and, as according to the theory of uniformly accelerated motions, these velocities are to each other in the sub-duplicate ratio of the said heights, whatever they may be, it is unquestionable that the permanent heads, under which the water has run out with the said velocities—must be to each other as the heights through which a falling heavy body would have acquired the same velocities at the end of the fall.

CHAPTER II.

ENQUIRY INTO THE STATE OF OVERFLOWN LIQUIDS IN RESERVOIRS.

XII.

Prop. I.—The surface of a liquid abandoned to the free action of gravity, and constituted in perfect equilibrium in the vessel of any form whatsoever, which contains it, is horizontal or perpendicular in all its points to the direction of gravity.

See the proof of this proposition in the work on hydrostatics.

XIII.

Prop. II.—Reciprocally, a liquid contained in a vessel, of any form whatsoever, and abandoned to the action of gravity, whose surface is at every point horizontal or perpendicular to the direction of gravity, is in perfect equilibrium.

XIV.

Corollary I.—Therefore, if a liquid contained in a vessel is but sensibly constituted in equilibrium, its surface will be only sensibly horizontal or perpendicular in all its points to the direction of gravity.

Corollary 11. And, reciprocally if the surface of a liquid contained in a vessel is sensibly horizontal all over, or perpendicular to the direction of gravity, the whole system will be sensibly in equilibrium.

XV.

Prop. III. The surface of the overflown water contained in reservoirs whence the liquid issues through orifices pierced in thin plates, fitted into the side or bottom, and wherein it is maintained during the flow at a uniform height above the centre of the orifices—remains always sensibly horizontal.

See Phenomenon 3 . § III of the foregoing chapter.

Corollary. I Therefore such a system of overflown water maintains itself during the flow sensibly in a state of equilibrium in the interior of the reservoirs (§XIV.)

Corollary II. But as in the interior of the reservoir, a motion must exist, in order that the efflux may be compensated for, there is not the shadow of a doubt (§ IV) but that the condition of this water is a mixed state which partakes both of continuous sensible rest and continuous motion.

XVI.

Prop. IV. This being so, to define the law and the natural symptoms proper to this state of overflow of the water in the interior of reservoirs.

Considering, in the first place, that in the permanent state we must suppose the efflux of the water through the orifice to be exactly equal to the supply at the upper part of the reservoir, it is unquestionable but that the outflow and the influx must take place simultaneously, otherwise, either, on the one hand, the outflow would not be uniform, or on the other hand, the upper limit of the overflow would not be constant. It is therefore indispensable that in the overflown liquid mass the passage of a quantity of water equal, neither more nor less, to that which issues through the opening or to that which comes in at the limit of the overflow, must take place and be verified at every instant; and as the whole body of the liquid is homogeneous, the water which comes in does, therefore, not pass by filtration through the overflown water, as it did through the oil (§ § VI, VII), but flows over immediately and spreads itself through the receiving water in the vicinity of the limit of the overflow, and it cannot reach the orifice to leave the vessel without the water which precedes it, and which is successively closer to this orifice having progressively made way for it. Hence the verification of this passage is effected by the successive translation and nearing to the orifice of the gradually anterior molecules. But on account of the perfect mobility of the water and the very delicate yieldingness of its parts, this effective interior motion cannot take place without the whole mass up to the exterior surface or limit of the overflow being affected by it. Hence there cannot exist in this mass absolute permanent rest nor permanent equilibrium between its parts—and consequently we cannot have an absolute permanent horizontality at the surface. Nevertheless, it is a fact (Phen. 3) that this surface maintains itself sensibly horizontal during the overflow, sensible equilibrium exists, therefore, between the parts of the water which is in the overflown state and consequently sensible rest in the whole system. But if there is, in this water, so constituted, neither an uninterrupted continuity of equilibrium nor of rest, because, contrary to fact, the surface should remain continuously and absolutely horizontal, nor yet an uninterrupted continuity of unstability, because, likewise, contrary to fact, the sensibly permanent horizontality of the surface could not subsist either, as in the imperfect fluids, it is necessary that in this singular condition of the water a perpetual succession of states of equilibrium and unstability should occur.

Hence, motion and rest, viz., unstability in the parts and return to equilibrum, must, necessarily, be successive. But, again, the horizontality of the surface and the egress through the orifice appear to be sensibly continuous. We must, therefore, conclude that the successive passages from rest to motion and *vice versa*, are, as much as can be so, a sudden operation of nature, instantaneous, very rapid. Therefore, the law and the systems proper to the overflown state of the water in the interior of reservoirs consist in the existence, within the overflown body of water, of a periodically variable condition, or of a particular kind of successive periodical passages from momentaneous rest to momentaneous motion, and from the latter again to rest—so that neither the rest of the system, from which results the sensibly continuous and permanent horizontality of the surface, nor the descensional motion which gives rise to the sensibly continuous and permanent reflux of the molecules towards the orifice—appear as if interrupted to the eye sight.

Whence, it is evident of what nature is this mixed state, as we have stated, (§ IV), which participates of rest and motion, and is as distinct from either the absolute state of rest or the absolute state of motion as these two states are distinct from one another, and unique of its kind. Q. E. D.

XVII.

Scholium.—There is, therefore, no definite or undetermined size of reservoir, nor any kind of vessel to which the law which we have just defined, is particularly limited. Whatever may be the form of the vessel wherein the liquid has an established, permanent surface, and whatever may be the opening through which it flows out uniformly, the liquid is always in a true state of overflow, and when in this state, neither the size nor form of the vessel, nor of the opening, enter into consideration. This is the characteristic property by which it may be recognized and distinguished from other states.

XVIII.

Prop. V.—The actual velocity of any molecule whatsoever, which traverses the mass of overflown water, during efflux, is always infinitely small.

For, as there is to be a successive passage from rest to descensional motion, and from the latter to rest, and so on, always alternatively, during the flow, all the small spaces described successively by a molecule will always intervene between two stationary periods, or periods of rest; and, consequently, there cannot be any descending molecule, in the act of falling which did not start from rest in the immediately preceding instant. But there is no determinate force which can impart, in an instant, a definite velocity to any body starting from rest. Wherefore, the actual velocity of any molecule whatever, descending through the mass of overflown water, will be, of necessity, infinitely small. Q. E. D.

XIX.

Corollary I.—If we suppose, therefore, a liquid which flows out with an infinitely small velocity, as soon as the efflux is permanently established, that sensible equilibrium exists between the parts of the system.

Corollary II.—In this state, therefore, which is that of the overflow, it is also quite evident that the law of sections, reciprocally proportional to the velocities, cannot strictly hold good in the overflown mass, as it does when the liquids move freely. For to make sure of such a law obtaining within the mass, it would be necessary either to use vessels of a definite form and size, which the nature of this state does not require, or to subordinate the momentary velocities of the molecules which traverse the mass to a law quite different from that which has really been shown to exist—which velocities are alternately extinguished at the renewal of equilibrium, and revived at the cessation of the same—and the alternative action being very persistent and imperceptible. Whence, it follows that the theories of the most illustrious hydrodynamicians on the motions of liquids issuing from orifices in reservoirs, are, perhaps wrongly founded on this law, which is necessarily excluded from the state of the overflow.

XX.

Scholium.—It is very difficult to reconcile a continuous acceleration of motion in the overflown water contained in reservoirs with the phenomena, and especially with those which show us openly that the velocity of the flow is due to the pressure of the water around the orifice, and never to the actual free fall from the upper limit of the overflow to the place of egress. The momentary stations, owing to which the sensible equilibrium of the parts is renewed at every instant, while they interrupt, at every instant, the downward course, preventing the velocity acquired by the molecules from being retained by them, and removing, at its origin, all acceleration—are, at the same time, those which give rise to an interior sensibly uniform but always elementary velocity being revived at every instant of rest, which constitutes an admirable economy of nature certainly well worthy of being developed and clearly pointed out, if I have succeeded in doing it properly.

XXI.

Scholium.—Hence, so long as the water contained in the vessels is in an overflown state, the system of the mixed state which we have defined, is preserved (§ XVI). and the velocity of the molecules can never be definite nor receive a determination. In order that this forever elementary velocity, and which, as we have said, always reappears after rest, may receive a determination, the water must pass from the overflown to the free state, which is truly the state wherein the water is not prevented from flowing with the velocity and in the direction of the motion which animates it, whether on account of the natural motion or owing to the forces by which it is solicited to move on.

XXII.

Scholium.—Because it has been demonstrated (§ XVIII) that the celerity $d\,c$ of any molecule whatever, passing through the mass of overflown water, is always indefinitely small, and that, besides, dynamics have shown to us that the initial velocity of a free point excited by any power whatever g, is proportional to the product $g\,d\,t$, of the power g, by the indefinitely small space of time $d\,t$, during which it remains applied to the same point, if any molecule whatever of overflown liquid solicited by the pressure around the orifice (§ IX, Coroll. III) becomes a free point, and that we call g the force or pressure which excites it, the velocity of this molecule in the instant $d\ t$, will be expressed by the product $g\ d\ t$. Therefore, this velocity which was $d\,c$, indeterminately in the state of overflow, becomes $g\,d\,t$, in the free state, and is determined by the equation $d\,c=g\,d\,t$. Hence, at whatever point of the overflown system this passage of the molecules from the state of overflow to the free state may occur, we will always have the equation:—

$$\text{(A)} \qquad dc=g\,d\,t=o.$$

XXIII.

Corollary I.—It is therefore demonstrated that equation (A) cannot hold good within the mass of liquids maintained at a uniform height in reservoirs in the actual and effective state of overflow such as they are in, and that it is applicable only to the free state; that is to say, when in overflown liquid masses, the passage from the former to the latter state takes place.

Corollary II.—And, therefore, remaining firm in the resolution to make no mental distinctions nor pliable hypotheses adapted to the laws of computation, but to conclude only what the phenomena or the rigorous reasoning lead us to conclude, we see, from what all that has been presented heretofore, that the motions which are commonly attributed to overflown liquids by hydrodynamicians are inexorably excluded from their midst.

XXIV.

Scholium.—No one perhaps, has come so near as Mr. D'Alembert to recognizing, in the liquids enclosed in vessels, the state of overflow which participates of the two states of motion and rest and which is yet essentially distinct from either. It is sufficient to examine the principles upon which he has based his theory of the motions of fluids to be convinced of this. And truly our equation (A) (§ XXII) which draws legitimately its origin from having taken cognizance of this state, might be used as a fundamental principle for solving all the problems of this illustrious geometrician, if a simple hydrodynamical speculation was my aim. But then a state of motion only would be assumed all through and not the actual state of overflow, which is the object aimed at, wherein this equation can in no way hold good. (§ XXIII).

We see by this, in what condition of things his theory agrees with the facts, viz., by supposing that the fluids are not in a state of overflow, but that they flow without the alternatives of descent or movement and equilibrium, which alternating actions destroy all acceleration and all continuity in the motions.

Scholium.—But for fear that by proceeding further with this enquiry which could easily be done, I might confound the objects, and render obscure the very clear ideas which we have just formed respecting the interior condition of liquids in the state of overflow, I will now explore, guided by the steps which have already been taken, the exterior movement of these liquids after they have passed from the overflown to the free state; and this will form the argument of the next chapter.

In the two remaining chapters (3rd and 4th) of his "Phisico-Mathematical Theory," Lorgna treats of the motions of liquids after they have emerged, as he says, from the state of overflow existing within reservoirs, through orifices pierced in their sides or bottoms, and of the contraction of the stream in horizontal, vertically descending and vertically ascending jets.

After explaining in what manner the liquid molecules issuing from orifices in reservoirs, wherein the liquids are maintained at a constant height above the centres of those orifices, are solicited by natural gravity and by the coaction of the pressures around the orifices combined, the author manages, by an ingenious train of reasoning, to fix the height due to the actual velocity in an orifice pierced in a thin plate at:

$$2\,\mathrm{H} \times 2\left(\frac{\sqrt{5}-1}{2}\right)^3 = 0{\cdot}472127\,\mathrm{H}$$

and arrives at:

$$2\,\mathrm{A}\,a^4\left(\frac{\sqrt{5}-1}{2}\right)^3 - y\left(x + 2\,\mathrm{A}\left(\frac{\sqrt{5}-1}{2}\right)^3\right) = o, \text{ or}$$

$$a^4\left({\cdot}472\,\mathrm{A}\right) - y^4\left(x + {\cdot}472\,\mathrm{A}\right) = o$$

for the equation of the hyperbolic conoïd of the contracted fluid vein—where A represents the permanent height of the fluid above the orifice, a the radius of this orifice, y the radius of a cross-section of the vein taken at any distance x, from the plane of the opening.

Putting $a = x = 1$, in the last equation, it becomes:

$${\cdot}472\,\mathrm{A} - y^4\left(1 + {\cdot}472\,\mathrm{A}\right) = o, \text{ whence:}$$

$$y = \frac{({\cdot}472\,\mathrm{A})^{\frac{1}{4}}}{(1 + {\cdot}472\,\mathrm{A})^{\frac{1}{4}}} = \left\{\begin{array}{l}\text{radius D E (Fig. 8) of circular cross-section of vein at a} \\ \text{distance of, say, } \tfrac{1}{2} \text{ diameter of the orifice, from its plane,}\end{array}\right\}$$

which is the formula of the hyperbolic conoïd of Newton.

The curve traced out by the extremities of the ordinates (y), calculated by means of this formula is, however, utterly at variance with the profile presented by the naturally contracted liquid vein, the contraction of which is much greater than that of the corresponding computed vein-form, as clearly shown by Venturi in the following table extracted from his "Experimental Enquiries."

Authors of Experiments.	Value of D E (Fig. 8) found by actual measurement.	Value of D E (Fig. 8) calculated by the preceding formula.
Poleni (de Castellis, § 35)	0 79	0·97
Michelotti; Sperim. Idraul., Tom. I., Exper 46; Tom. II., Exper. 4.	0·80	0·99
Bossut (Hydrodyn, Art. 437, Exper. 5)	0·818	0·99
Venturi, with 35 inches charge and a horizontal circular orifice, 18 old French lines=1·5985 English inches in diameter	0·798	0·984

"It is evident," says Venturi, "that the contraction of the vein, as found by "experiment, is incomparably greater than can be produced by the acceleration of "gravity, even in descending streams. But what can we say of horizontal and "ascending jets, in which assuredly the action of gravity does not take place, but in "which, nevertheless, the contraction is observed nearly in the same manner as in "descending currents? The contraction of the stream is therefore very different "from the Newtonian hyperboloid."

Venturi further adds: "Desirious of proving that the vein does not possess the "whole velocity arising from the height of the fluid above the centre of the orifice, "Lorgna relates the experiments of Kraft,* which are not applicable to the question, "because they were made with cylindrical pipes, and we have seen that such pipes "always destroy part of the velocity of the fluid; consequently we cannot establish "any rule from them which shall apply to orifices through thin plates.† He wishes "not to determine the velocity of ascending jets by the height to which they rise, "because he is apprehensive that the preceding part of the stream or jet is urged, "and supported by the succeeding part nearly to the height of the charge. Never-"theless, if we intercept the jet all at once, the last portions of water fly to the same "height as those which preceded them, without having any continued column of the "fluid below to follow and support them; these last portions must, consequently, "have received, at their passage through the orifice, all the velocity which was "necessary to raise them nearly to the surface of the fluid in the reservoir."

*Acta Petrop. vol. VIII.

†Torcelli took notice of this difference at page 168 of his works, "*quoties cumque autem aqua per tubem latentem decurrens per angustins transire debuerit, falsa omnia reperies.*"

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